Experimental Validation of Gravitational Acceleration Measurement Using Kater's Reversible Pendulum

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Abstract - *The measurement of gravitational acceleration is a fundamental experiment in physics education, providing students with essential insights into classical mechanics and experimental techniques. This study presents an experimental validation of gravitational acceleration measurements using a low-cost Kater's reversible pendulum developed for undergraduate laboratory applications. The apparatus consists of two knife-edge pivots and adjustable masses, enabling nearly identical oscillation periods about both pivot points. Oscillation periods were measured for various positions of the movable mass, while sensor-based measurements were used for verification. The equal-period condition was determined analytically from the experimental data and subsequently used to calculate the local gravitational acceleration. The measured value was (9.768 ± 0.012) m* s^{-2}*, with the measurement uncertainty estimated using partial-derivative error propagation based on the instrumental uncertainties in length and time measurements. The measured value agrees well with the accepted local gravitational acceleration, with a relative deviation of approximately 0.13%. These findings demonstrate that the developed Kater's reversible pendulum provides reliable measurements of gravitational acceleration and is suitable for undergraduate physics laboratories. The results also indicate that accurate determination of gravitational acceleration primarily depends on achieving equal oscillation periods and precisely determining the effective pendulum length.*

Keywords*: Kater's Reversible Pendulum; Physical Pendulum; Gravitational Acceleration*

INTRODUCTION

The acceleration due to gravity (g) is a fundamental physical quantity whose magnitude varies across the Earth as a function of latitude, altitude, and the planet's oblate shape. In physics education, understanding this concept is vital as it underpins classical mechanics, fluid dynamics, and planetary science. In laboratory settings, measurements of g are predominantly conducted using simple mathematical pendulums. However, the theoretical model of an idealized simple pendulum introduces a significant gap between idealized assumptions and actual laboratory performance. An ideal mathematical pendulum assumes a completely massless string and a point-mass bob (Halliday, Resnick, & Walker, 2010). In physical reality, strings possess non-negligible mass, and bobs occupy geometric volume, leading to persistent systemic errors in traditional experiments (Tipler, 1998).

To bridge this gap, a physical pendulum approach must be utilized. Unlike mathematical models, a physical pendulum treats the apparatus as an extended rigid body, accounting for its total mass, geometric dimensions, and internal moment of inertia via the Parallel Axis Theorem (Serway & Jewett, 2014). A highly sophisticated instrument based on this principle is Kater's Reversible Pendulum, which eliminates the need to pinpoint the exact center of mass or calculate the internal moment of inertia of complex geometries (Alonso & Finn, 1992). The system relies on the principle that if a rigid body is swung from two opposing, asymmetrical knife-edge pivots and its positions are adjusted until the oscillation periods from both pivots are identical, the distance between the two pivots can be treated as the effective length of an equivalent simple pendulum.

Recent developments in physics education have focused on improving the accuracy and accessibility of gravitational acceleration experiments through low-cost experimental apparatus and digital measurement techniques. Several studies have demonstrated that integrating optical encoders, microcontroller-based timing systems, and other sensing technologies can improve the precision and repeatability of pendulum experiments while remaining suitable for undergraduate laboratories (Carvalho et al., 2025; Bernal et al., 2025; Wadhwa et al., 2025). Nevertheless, many of these systems still rely on specialized electronic components or commercially manufactured equipment that may not be readily available in institutions with limited laboratory resources.

Although Kater's reversible pendulum has long been established as a standard instrument for determining gravitational acceleration, opportunities remain to improve its accessibility for undergraduate laboratory instruction, particularly in institutions with limited laboratory resources. Most previous studies have focused on commercially manufactured apparatus or sophisticated digital instrumentation to improve measurement accuracy. In contrast, this study emphasizes the experimental validation of a locally developed Kater's reversible pendulum fabricated from readily available materials. The developed apparatus is validated by comparing the measured gravitational acceleration with the accepted local gravity value and by assessing the consistency of the oscillation period measurements using a photogate sensor. This approach aims to provide a reliable and cost-effective alternative for gravity experiments in undergraduate physics laboratories. Therefore, this study aims to experimentally validate the performance of a locally developed Kater's reversible pendulum for determining local gravitational acceleration. The study also evaluates the agreement between the measured and expected gravitational acceleration values, as well as the consistency of the experimental measurements through sensor-based verification.

RESEARCH METHODS

This experimental quantitative research was conducted at the Basic Physics Laboratory, Physics Education Study Program, Universitas Sebelas Maret, Surakarta. The research method is entirely based on physical experimentation rather than theoretical derivations.

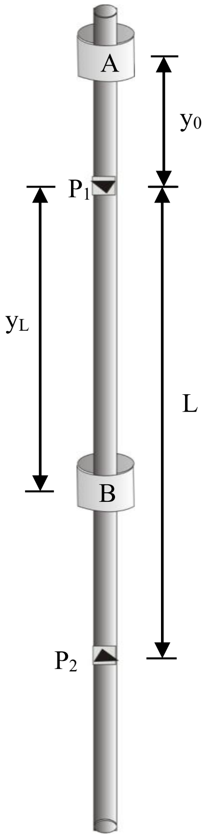

Figure 1. Design of the Kater's Reversible Pendulum

The custom apparatus consists of a cylindrical stainless steel pendulum rod with a diameter of 10 mm and a length of 750 mm, serving as the main body in the Fig. 1. Two asymmetrical iron knife-edge pivots with a nickel-plated finish were mounted firmly on the rod at a fixed distance (L = 0.5 m) from one another. To introduce the necessary asymmetry required for Kater's principle, two cylindrical nickel-plated brass weights (0.2 kg each) were attached to the rod; one mass was locked at a fixed position near the top pivot, while the second mass functioned as an adjustable sliding mass (y_L). The complete pendulum assembly was suspended vertically from a robust table clamp and steel support stand assembly.

The implementation procedure began by systematically shifting the position of the sliding mass (y_L) along the rod, measured directly from the first knife-edge pivot. The distance was varied from 5 cm to 45 cm using an increment interval of 5 cm. For every specific interval of y_L, the pendulum was displaced by a small angle ($\theta < 5°$) to maintain simple harmonic motion characteristics. The time required to complete exactly n = 10 full oscillations was recorded. This measurement process was performed twice for each distance interval: first in the upright configuration (Pivot 1 as the rotation axis, yielding time t_a and period T_a) and second in the inverted configuration (Pivot 2 as the rotation axis, yielding time t_b and period T_b).

Time intervals were tracked using a high-precision digital stopwatch with a precision threshold of 0.001 seconds. To cross-verify the temporal accuracy and minimize human delay, a secondary digital counter timer system linked with an infrared photogate sensor array was deployed intermittently. The stopwatch measurements were used as the primary dataset for determining the oscillation periods and calculating gravitational acceleration, whereas the photogate sensor measurements were employed solely for verification purposes to assess the consistency of the manually obtained results. Spatial distances were evaluated with a standard steel laboratory ruler possessing a minimum scale division of 1 mm. For each sliding-mass position, the oscillation time was measured five times for both suspension points. The oscillation period reported in this study represents the average of the five repeated measurements.

Data analysis was performed by plotting the relationship between the oscillation periods (T_a) and (T_b) and the sliding mass position (y_L). The measured periods from both suspension points were then compared for each sliding-mass position to identify the condition at which the difference between (T_a) and (T_b) was minimal, representing the equal-period condition ($T_a \approx T_b$). The effective period (T) was calculated as the average of the corresponding (T_a) and (T_b) values and subsequently used, together with the fixed pivot distance (L), to determine the local gravitational acceleration and its associated uncertainty using the partial differential method.

RESULTS AND DISCUSSION

Results

The constant parameters maintained throughout the experimental process were a fixed distance between the two knife-edge pivots (L) of 50 cm (0.5 m), a geometric constant π value of 3.14, and an oscillation count (n) of 10. The comprehensive data gathered during the physical experimentation, which documents the relationship between the sliding mass positions and the corresponding oscillation periods for both upright and inverted axes, is structured in Table 1.

Table 1. Sliding Mass Position and Oscillation Periods of Both Pivots

No	yL (cm)	T_a (s)	T_b (s)	$\bar{T}$(s)
1	5	1.43292	1.48108	1.457
2	10	1.42052	1.45086	1.43569
3	15	1.40834	1.41144	1.40989
4	20	1.39682	1.39188	1.39435
5	25	1.39976	1.3947	1.39723
6	30	1.40378	1.39826	1.40102
7	35	1.40972	1.41052	1.41012
8	40	1.4211	1.42052	1.42081
9	45	1.44684	1.45646	1.45165
Sum				12.77776
Average				1.419751

Figure 2 illustrates the variation of the oscillation periods measured about the two suspension points as the sliding-mass position changes. The condition corresponding to nearly equal oscillation periods was identified by comparing the measured values of T_a and T_b at each sliding-mass position, rather than from the graphical representation itself.

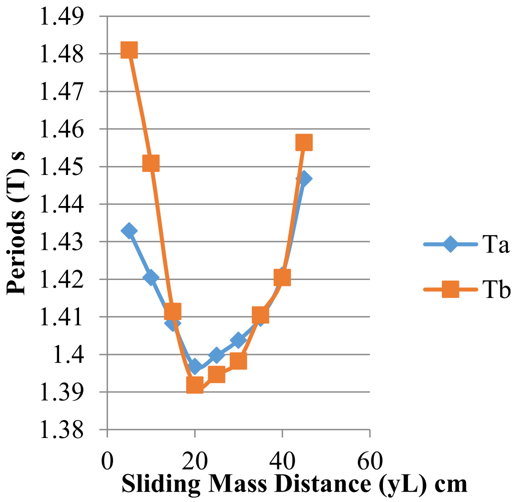

Figure 2. Oscillation Periods T_a and T_b as a Function of Sliding Mass Distance y_L

Among the investigated sliding-mass positions, the smallest difference between T_a and T_b was obtained at y_L= 40 cm, where T_a =1.42110 s and T_b=1.42052 s. Each reported oscillation period represents the average of five repeated measurements performed at the corresponding sliding-mass position. This position was therefore considered to satisfy the equal-period condition ($T_a \approx T_b$) required for the Kater's reversible pendulum method. The effective oscillation period was calculated as the average of the corresponding values, yielding T=1.42081 s. Using the fixed distance between the knife-edge pivots (L=0.5 m), the local gravitational acceleration was determined to be 9.76826 m/s^2.

The evaluation of instrumental limits, established as half of the smallest scale division of the measuring tools, provides an absolute spatial uncertainty (ΔL) of 0.0005 m and a temporal uncertainty (ΔT) of 0.0005 seconds. Processing these values through the partial differential error propagation formula yields an absolute gravitational uncertainty (Δg) of 0.01195 m/s². This mathematical analysis determines a relative experimental error margin of approximately 0.13%, indicating that the measured gravitational acceleration is in close agreement with the accepted local gravity value. Consequently, the localized gravitational acceleration value is formally reported as (9.768 ± 0.012) m/s².

The measured local gravitational acceleration, g = (9.768 ± 0.012) m/s², was compared with the reference gravity value for the Surakarta region estimated using the GRS80 normal gravity model, which is consistent with the geodetic reference framework adopted in the Earth Gravitational Model 2008 (EGM2008). For the latitude of the experimental location (approximately 7.56° S), the reference gravity value is approximately 9.781 m/s². The difference between the experimental and reference values is approximately 0.013 m/s² (about 0.13%), indicating good agreement between the measured result and the accepted local gravity value. The remaining discrepancy may be attributed to manual timing, slight misalignment of the knife-edge pivots, air resistance, and other unavoidable laboratory uncertainties.

Discussion

The empirical findings generated by this research validate the operational capacity and scientific reliability of the custom-constructed Kater's Reversible Pendulum. As observed in the experimental data, shifting the sliding mass away from the primary pivot causes the oscillation periods to initially drop until they reach a clear minimum threshold near the 20 cm position, after which further outward movement forces a steady, continuous increase in period values. This particular behavior confirms the foundational laws of rigid body dynamics, which state that the square of an object's oscillation period (T^2) is directly proportional to the variation of its rotational axis relative to its shifting center of mass (Serway & Jewett, 2014).

In the context of physical pendulum optimization, the dynamic shift in these period curves is tightly governed by the structural mass distribution. Research by Stewart (2009) on the mathematics of Kater's apparatus emphasizes that the intersection point where $T_a \approx T_b$ is highly sensitive to the precise positioning of the non-symmetrical weights. When the sliding mass is adjusted, it systematically alters the distance between the pivot and the center of mass (h), thereby modifying the local radius of gyration. This structural behavior aligns with historical geodetic experiments, where achieving a clean convergence between the two opposing periods serves as the absolute condition to isolate the effective pendulum length from internal geometric complexities (Alonso & Finn, 1992).

The measured local gravitational acceleration, g = 9.768 m/s², differs by approximately 0.013 m/s² from the reference gravity value for the Surakarta region (approximately 9.781 m/s²). This small difference corresponds to about 0.13%, indicating good agreement between the experimental result and the accepted local gravity value. The observed discrepancy is consistent with known geographic variations in gravitational acceleration, which depend on latitude, Earth's rotation, and the oblate shape of the Earth (Tipler, 1998).

Beyond geographic and astronomical parameters, the minor downward shift in the recorded acceleration can also be analyzed through the lens of ambient experimental constraints. Investigative studies conducted by Kopac, Bochnicek, and Pecen (2018) demonstrate that open-air physical pendulums are subject to systematic buoyancy and air resistance effects. The ambient air density exerts a minute upward buoyant force on the oscillating brass weights, effectively reducing the net gravitational torque acting on the rigid body. Furthermore, aerodynamic drag forces acting along the extended 750 mm stainless steel rod introduce a continuous energy dissipation channel. As detailed by Cross (2021) in his analysis of pendulum damping mechanisms, even minor air resistance and microscopic friction at the knife-edge support boundary slightly lengthen the observed oscillation periods (T). In addition, measurements obtained using the photogate sensor showed good consistency with the manually measured periods, providing further support for the validity of the experimental results. Since the period appears in the denominator of the primary gravitational formula ($g = 4\pi^2 L/T^2$), any slight systematic inflation in T naturally translates into a marginally lower calculated value for g.

Furthermore, these findings align closely with previous experimental developments recorded in physics education literature. Similar studies focusing on low-cost physical pendulum variations have demonstrated that carefully constructed rigid rods can achieve high levels of precision, provided that error analysis frameworks are strictly applied (Alonso & Finn, 1992). The minor errors encountered during data collection stem primarily from human reaction delays during the manual synchronization of the stopwatch at the exact start and end of the tenth oscillation cycle, alongside visual resolution limitations when reading the millimeter markings on the steel ruler. Although minor uncertainties were present during data collection, the measured gravitational acceleration remained in good agreement with the accepted local gravity value, supporting the reliability of the developed apparatus. These results demonstrate that the developed apparatus can provide reliable measurements of gravitational acceleration and may serve as an accessible alternative for university-level mechanics and laboratory instruction.

CONCLUSION

This study successfully developed and experimentally validated a laboratory-scale Kater's reversible pendulum constructed from a stainless-steel rod and nickel-plated brass weights. The experimental results showed that the oscillation periods measured about the two suspension points varied systematically with the position of the sliding mass, leading to an equal-period condition that enabled the determination of local gravitational acceleration. Using the measured oscillation period and the fixed distance between the two knife-edge pivots, the local gravitational acceleration was determined to be $g = (9.768 \pm 0.012)\ m/s^2$. The measured value differs by only approximately 0.13% from the accepted local gravity value for the Surakarta region, demonstrating good agreement between the experimental result and the reference value. These findings indicate that the developed Kater's reversible pendulum can provide reliable measurements of gravitational acceleration and may serve as an accessible experimental apparatus for undergraduate physics laboratories. Future improvements may include the use of fully automated digital timing systems, higher-precision length measurements, and improved mass-position adjustment mechanisms to further reduce experimental uncertainties.

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