ANALYSIS OF STUDENTS’ MATHEMATICS COMMUNICATION ABILITY BASED ON COGNITIVE STYLES AND MATHEMATICAL KNOWLEDGE

Syahrul Azmi*, Baidowi, Nurul Hikmah, Ratna Yulis Tyaningsih, and Eka Kurniawan
Mathematics Education Department, FKIP University of Mataram, Mataram, Indonesia
*Email: syahrulazmi.fkip@unram.ac.id

Received: February 15, 2021. Accepted: March 6, 2022. Published: March 26, 2022

Abstract: This study aims to describe the mathematical communication skills of mathematics education students based on their cognitive styles and mathematical knowledge. This research is a qualitative approach. Data were collected by giving the MFFT test to the students to measure their cognitive styles and essay tests for determining their mathematical communication skills. Data were analyzed by comparing the scores of cognitive styles, mathematical communication, and their mathematical performances. The result shows that students with reflective cognitive styles perform better on mathematical communication than impulsive cognitive styles. Moreover, their mathematical knowledge significantly affects their abilities in solving mathematical communication tests.

Keywords: Mathematics Communication Ability, Cognitive Style, Mathematical Knowledge

INTRODUCTION

In the Regulation of the Minister of National Education No. 22 2006, regarding the standard of content for learning mathematics, it is stated that one of the objectives of learning mathematics is so that students can communicate ideas with symbols, tables, diagrams, or other media to clarify situations or problems. The ability to communicate with symbols, tables, graphs, or media is part of mathematical communication skills [1]. Communication skills in learning mathematics need to be developed among every learner because mathematics is not just a thinking tool for finding patterns, solving problems, or drawing conclusions. But mathematics is also a social activity in learning mathematics, mathematics as a vehicle interaction between students and communication between teachers and students [2]. In NCTM [3], it is stated that mathematical communication skills need to be built in students so that they can: (1) model situations verbally, in writing, pictures, graphs, and algebraically; (2) reflecting and clarifying in thinking about mathematical ideas in various situations; (3) develop an understanding of mathematical ideas including the role of definitions in mathematics, (4) use reading, listening, and seeing skills to interpret and evaluate mathematical ideas, (5) examine mathematical ideas through conjectures and convincing reasoning, and (6) understand the value of notation and the role of mathematics in the development of mathematical ideas.

In general, mathematical communication skills are divided into two, namely verbal mathematical communication skills and written mathematical communication skills. Oral mathematical communication skills are defined as students’ reading, listening, discussing, and explaining abilities. Meanwhile, written mathematical communication skills are defined as the ability of students to express mathematical ideas in real-world phenomena through graphs or pictures, tables, algebraic equations, or in everyday language [4].

Based on the results of research conducted by Azmi, Hayati, Hapipi, and Triutami in 2020, in the calculus II course, it was found that students who took calculus II courses already had fairly good mathematical communication skills. Of the 31 students who took the mathematical communication ability test, there were 28 who completed it. The highest score is 100, and the minimum score is 30. Classically, the average value achieved is 75.63 [5].

For students’ mathematical communication skills to be even better, they must be followed by the prerequisite skills that meet the requirements. In this case, the prerequisite ability is the student's initial ability. Students’ initial ability is a determining factor in the success of learning mathematics [6]. The initial ability of students/students is the ability that has been possessed by students/students before participating in the learning that will be given. This initial ability (entry behavior) describes the readiness of students/students to receive lessons delivered by teachers/lecturers. A person's success in a class or education also depends on the person's enthusiasm. There are two kinds of readiness for this child, their mental development is ready, and they already have prerequisite knowledge [7].

In solving a problem, students need thinking to solve the problem. Differences in thinking characteristics of students will solve problems in different ways so that the learning outcomes achieved by each student are also different. Each individual has unique characteristics that other individuals do not share when solving problems. One of the differences in student characteristics can be seen from their cognitive style.

Some notions of cognitive style (cognitive style) include Martines & Kaufmann, who state that "cognitive style refers to the manner or way of processing information. Cognitive style refers to the
attitude or way a person processes information [8]. Cognitive style is how a person processes, stores, or uses the information to respond to a task or various environmental situations [9]. Meanwhile, Witkin, Ortmann, Raskin, & Karp (1971) state that: “cognitive style is defined as the characteristic, self-consistent mode of functioning that an individual shows in his perceptual and intellectual functioning” [10]. Based on the above definition, it can be concluded that cognitive style is an individual characteristic in receiving and processing information based on their perception and intellect.

Based on the description above, it can be concluded that mathematical communication skills are essential to be mastered by prospective teacher students because concepts in mathematics must be clearly communicated to their students later, both verbal and written mathematical communication. Therefore, it is necessary to examine the mathematical communication skills of students of Mathematics Education FKIP Unrau in more depth.

So, the purpose of this study was to describe the mathematical communication skills of students in mathematics education in online learning in terms of cognitive style and initial abilities.

**RESEARCH METHOD**

This type of research is a qualitative descriptive study. This type of research describes students’ mathematical communication skills in terms of their initial abilities and cognitive style.

The data in this study were taken using a test technique, namely the MFFT test for cognitive style and the mathematical communication ability test. The initial ability category is seen based on student scores in the Basic Mathematics course in the previous semester.

The communication ability indicator used in this study adopts the communication ability indicator according to NCTM (2000) and Ansari (2016). For data analysis techniques, student answer sheets to mathematical communication skills test questions were collected, then analyzed using the following scoring guidelines (table 1).

### Table 1: Scoring Guidelines for mathematical communication skills tests

<table>
<thead>
<tr>
<th>No.</th>
<th>Completeness Criteria</th>
<th>Indicators</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Explains mathematical ideas, situations and relations in writing with graphic images, and algebraic analysis.</td>
<td>No answer</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Make a description of the answer with algebraic analysis and graphs, but some are correct and correct</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Make a description of the answer with algebraic and graphic analysis, but some are true and correct</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Make a description of the answer with algebraic analysis and graphs, but most of it is true and correct</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All answers are correct and correct</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Connect real objects, pictures, tables, and diagrams into mathematical ideas.</td>
<td>No answer</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Connecting between the given pictures into a mathematical idea, but some of them are correct and correct</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Connecting the pictures given into a mathematical idea, but some are true and correct</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Connecting between the pictures given into a mathematical idea but mostly true and correct</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All answers are correct and correct</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>State everyday events in symbols mathematical</td>
<td>No answer</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Make a mathematical model based on real-world problems and solve but some are true and correct</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Make a mathematical model based on real-world problems and solve but some are true and right</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Make a mathematical model based on real-world problems and solve them, but most of it is correct and correct</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All answers are correct and correct</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

The cognitive style is calculated based on the median time used (t) and the median correct answer (f). Students are said to have a reflective cognitive style if t >= median time and many correct answers f >= 7. For impulsive cognitive style, if t < median time and many correct answers f < 7.

The initial abilities of grade 2C students are grouped into three categories, namely groups of
students with high (T), moderate (S), and low (R) initial ability categories.

Guidelines for determining initial ability categories are based on the table 2:

<table>
<thead>
<tr>
<th>Interval Value</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>72 x 100</td>
<td>High</td>
</tr>
<tr>
<td>56 x &lt; 72</td>
<td>Medium</td>
</tr>
<tr>
<td>x &lt; 56</td>
<td>Low</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION
Student Cognitive Style Test Results

To determine the student’s cognitive style, what is observed is the time used to complete the test (t) and the number of correct answers. Based on the length of time used by each student, median time (t) = 14.17 minutes, and the median correct answer (f) = 7. Students are said to have a reflective cognitive style if t > 14.17, and many correct answers are f > 7. For impulsive cognitive style, if t < 14.17 and many correct answers f < 7, while the other two types of cognitive style are fast accurate if t < 14.17 and many correct answers f > 7 and slow inaccurate if t > 14.17 and many correct answers f < 7. Based on the results of the cognitive style test on 27 students of mathematics education class 2C, the data on cognitive style were obtained as follows:

Table 2. Students’ initial ability categories

<table>
<thead>
<tr>
<th>Interval Value</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>72 x 100</td>
<td>High</td>
</tr>
<tr>
<td>56 x &lt; 72</td>
<td>Medium</td>
</tr>
<tr>
<td>x &lt; 56</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table 3. Data on the distribution of cognitive styles of students of the Mathematics Education Study Program class 2C Year 2020/2021

<table>
<thead>
<tr>
<th>Categories</th>
<th>Many of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflective</td>
<td>5</td>
</tr>
<tr>
<td>Impulsive</td>
<td>10</td>
</tr>
<tr>
<td>Fast Accurate</td>
<td>4</td>
</tr>
<tr>
<td>Slow Inaccurate</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>27</td>
</tr>
</tbody>
</table>

Results of Grouping Students’ Initial Ability

Table 3. Initial Abilities of Mathematics Education Students Class 2C in 2020/2021

<table>
<thead>
<tr>
<th>Category</th>
<th>Many Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>9</td>
</tr>
<tr>
<td>Medium</td>
<td>14</td>
</tr>
<tr>
<td>Low</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>27</td>
</tr>
</tbody>
</table>

Description of students’ mathematical communication skills based on cognitive style

a. Description of students’ mathematical communication skills with reflective cognitive style.

The picture below is an example of a student’s answer with a reflective cognitive style.

Figure 1 Students’ answers with reflective cognitive style (R)

1. \( f(x) = x^2 \).
The function \( f(x) \) is a function that is continuous at \( x = 0 \) but not differentiable at \( x = 0 \).

Chart:

- Obviously at \( x = 0 \), \( f(x) = 0 \), it means that \( f(0) \) exists.
- \( \lim_{x \to 0} f(x) = \lim_{x \to 0} \) exists.
- \( \lim_{x \to 0} x^2 = \lim_{x \to 0} x^2 \)
- \( 0 = 0 \)
- Since \( \lim_{x \to 0} f(x) = \lim_{x \to 0} f(x) \), then \( f(x) = 0 \), meaning there is
  - It is clear that \( \lim_{x \to 0} f(x) = 0 \) ( ) is the same as \( f(x) = 0 \), or
  - \( \lim_{x \to 0} g(x) = g(0) \)
  - So \( g(x) = x^2 \) is continuous at \( x = 0 \)

Continued no. 1

Prove that \( f(x) = x^2 \) is not differentiable at \( x = 0 \)

\[
\begin{align*}
f’(0) &= \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} \\
&= \lim_{h \to 0} \frac{(0 + h)^2 - (0)^2}{h} \\
&= \lim_{h \to 0} \frac{h^2}{h} \\
&= \lim_{h \to 0} h = \lim_{h \to 0} 1 = 1 = \infty
\end{align*}
\]

At \( x = 0 \) the value of \( f(0) \) is infinite, meaning that \( f(x) \) at \( x = 0 \) does not exist / is undefined. So at \( x = 0 \), \( f(x) \) is not differentiable.
In Figure 1, it can be seen that students began to answer by drawing graphs of the given function. Next, students begin to analyze by checking the continuity of the function \( f(x) = x^{2/3} \) at \( x = 0 \), and then students begin to check whether the function is differentiable in \( x = 0 \) using the definition of the derivative. It can be seen that the student’s work steps are traced and have been quite thorough in answering even though, in the result of the division of 1 by 0, the answer is wrong.

b. Student’s Mathematical Communication Ability based on impulsive cognitive style

Based on Figure 2, it can be seen that students with the type of impulsive cognitive style immediately look for the derivative of \( f(x) = x^{2/3} \) at \( x = 0 \) and find the function’s derivative at 0 does not exist.

**Students’ mathematical communication ability is based on initial**

a. Ability Student’s mathematical communication ability based on High initial ability (T)

The following is presented the results of the students’ mathematical communication ability test answers with the category of High initial ability (T), on the 1st indicator, namely Explaining mathematical ideas, situations and relationships writing with real objects, pictures, graphics, and algebra.

Students can explain in writing that a continuous function is not necessarily differentiable. This student’s answer is also clarified with a graphic sketch to support his answer, presenting with a picture that a graph with sharp corners is not differentiable at that point.

b. Students’ mathematical communication skills based on moderate initial ability (S)

Students’ mathematical communication skills with a moderate initial ability (S). In solving problem number 1, they are unclear in expressing ideas using pictures and their algebraic analysis. It can be seen in the description of the following answers:

Based on Figure 4, it is known that the student can only show that the function is continuous and does not provide an algebraic explanation of what causes the function to be undifferentiated. The graph depicted is only a simple graphic image without further explaining the graph.

Based on the description in figures, table 4 is a summary of the analysis results of students’ mathematical communication skills based on their cognitive styles and initial.

Table 4 above shows the differences between students in completing tests of mathematical communication skills. One of the reasons for this difference is the difference in the characteristics of their thinking. Differences in thinking characteristics of students will cause them to solve and solve problems in different ways so that the learning outcomes achieved by each student are also different. Each individual has distinctive characteristics that other individuals do not share. One of these distinctive characteristics can be seen based on their cognitive style. Cognitive style is how a person processes, stores, or uses the information to respond to a task or to various environmental situations [6].
1. The function \( f(x) = x^\frac{2}{3} \) is a function that is continuous at \( x=0 \) but not differentiable at 0. Give an explanation with a graphic image of the function why this is so.

Solution:

Prove that \( f(x) = x^\frac{2}{3} \) is not differentiable at \( x=0 \)

\[
\frac{df}{dx} = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \to 0} \frac{h^{\frac{2}{3}} - (0)^{\frac{2}{3}}}{h}
\]

\[
= \lim_{h \to 0} \frac{h^\frac{2}{3}}{h} = \lim_{h \to 0} h^{\frac{2}{3}-1} = \lim_{h \to 0} h^{-\frac{1}{3}}
\]

\[
\lim_{h \to 0} \frac{1}{h^{\frac{1}{3}}} = \text{undefined}
\]

Since the value \( f'0 \) is undefined, \( f(x) = x^\frac{2}{3} \) is not differentiable at \( x=0 \)

Graphic Drawing:

Since the graph has sharp corners, the function is not differentiable at \( x = 0 \)
Table 4. Summary of the results of the analysis of mathematical communication skills of mathematics education students  

<table>
<thead>
<tr>
<th>No</th>
<th>Minimum Completeness Criteria Indicator</th>
<th>Cognitive style</th>
<th>Initial Ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Explaining mathematical ideas, situations and relations in writing with real objects, pictures, graphs, and algebra</td>
<td>RF = 4, Im = 2</td>
<td>T = 4, S = 2, R = 1</td>
</tr>
<tr>
<td>2</td>
<td>Connecting real objects, pictures, tables, and diagrams into ideas mathematics.</td>
<td>RF = 3, Im = 2</td>
<td>T = 4, S = 2, R = 1</td>
</tr>
<tr>
<td>3</td>
<td>Expressing everyday events in symbols mathematical</td>
<td>RF = 2, Im = 2</td>
<td>T = 3, S = 1, R = 0</td>
</tr>
</tbody>
</table>

Based on Table 4, it is known that students with reflective cognitive style have better mathematical communication skills than students with impulsive cognitive style. Students with reflective cognitive style can fulfill two indicators of mathematical communication skills well, while the 3rd indicator is still not appropriately fulfilled. Students with reflective cognitive style are more thorough and consistent in providing answers and analysis and conclusions on their answers. It can be seen from the answer (figure 1) that students who have a reflective cognitive style have been able to meet the indicators of mathematical communication skills. Students can explain ideas, situations and mathematical relationships in writing with real objects, pictures, graphs, and algebra.

It can be seen that students with reflective cognitive style have done algebraic analysis and have also made graphs correctly and entirely according to the questions given. Likewise, in question number 2, students with a reflective style have used terms, mathematical notations, and structures to present ideas and describe the relationship between variables, although it is not entirely correct. Students can express mathematical arguments in writing, as well as draw visually. As for question number 3, students have been able to interpret contextual story questions in the form of pictures, but they are not appropriate in making the equations of the cost function. However, it seems that students are pretty thorough in analyzing and more detailed in describing answers. Both studies show that students with reflective cognitive style tend to be more comprehensive and consider many possibilities before answering or solving questions so that the answers produced are more detailed and informative [11-12]. The description of students' answers with reflective cognitive style shows that these students have met the characteristics of mathematical communication [13].

Meanwhile, students with impulsive cognitive style tend to answer questions incorrectly; this can be seen from the answer to question number 1, where the student is less precise in visualizing the functions given in the graphic form. It shows that the student has not met the first indicator of mathematical communication skills, namely explaining mathematical ideas, situations and relations in writing with real objects, pictures, graphs, and algebra.

Likewise, in question number 2, students are not precise in connecting the images given in the problem into mathematical ideas. It can be seen from the form of examples made by students where
students are still wrong in assuming the length of the box. It shows that students have not fully met the second indicator of communication skills, namely connecting real objects, pictures, tables, and diagrams into mathematical ideas. As for question number 3, students are still not right in interpreting sentences in problems of everyday events into mathematical sentences. It can be seen from the equation made by students to express the circumference of the cage. It shows that students have not met the third indicator of communication skills, namely stating daily events in language or mathematical symbols.

Of the three indicators of mathematical communication skills, students with impulsive cognitive style only fulfill some of the indicators. If it is observed from the description of the student's answer, the student is in a hurry to answer without considering other aspects of the answer. As a result, the answers given by students are inaccurate. It follows the characteristics of students with an impulsive cognitive style: fast in answering problems, but not / less careful, so the answers tend to be wrong [12]. Students with an impulsive cognitive style tend to be in a hurry, so they often experience misunderstandings about the information obtained. It resulted in error results and conclusions generated [11]. Students with an impulsive cognitive style are inaccurate and incomplete in solving the given problem [14].

Based on the description above, it can be concluded that students' mathematical communication skills are caused by differences in processing information in the brain. The delivery of students’ mathematical ideas depends on their cognitive style [15]. Students with a reflective cognitive style are more precise in expressing their ideas and ideas in solving a given problem than students with an impulsive cognitive style. Overall mathematical communication skills of subjects with reflective cognitive style, both male and female, were better than subjects with impulsive cognitive style, both male and female [16].

Further research aims to analyze students' mathematical communication skills based on their initial abilities. Students' initial ability is the ability that students already possess before participating in the learning that the teacher will give. This initial ability describes students' readiness to receive a material [17]. The initial ability referred to in this study is the student's mastery of the prerequisite material for the calculus course, namely the Basic Mathematics course. In introductory mathematics courses, the material is closely related to calculus material, so students' mastery of calculus courses is based on students' materials ability in introductory mathematics courses. The learning process that connects previously studied material/knowledge with new material/knowledge makes learning meaningful. The main factors that influence meaningful learning are the existing cognitive structure, stability, clarity of expertise in a field of study, and time [18]. Students with a high initial ability (T) have met the three indicators of mathematical communication ability when viewed from students' initial ability. However, the third indicator has not been fulfilled perfectly. Students with the moderate initial ability only fulfill part of each indicator of mathematical communication ability. Meanwhile, students with low initial abilities have not been able to meet the three indicators of mathematical communication skills.

Students with low initial abilities will find it more difficult to acquire new knowledge and relate it to their previous knowledge. Meanwhile, students with high initial ability will be more receptive to further information and connect it to pre-existing details [19]. Once students understand the subject matter, it is influenced by the initial abilities they already have. Students' low level of knowledge from the previous material or prerequisite material for the following material will cause students to have difficulty continuing and understanding the following material [20]. There is an increase in students' mathematical communication skills with high initial abilities, while students with moderate and low initial abilities do not experience an increase in mathematical communication skills [21-23].

**CONCLUSION**

Based on the results of the study, it can be concluded several things as students with reflective cognitive style are more precise in expressing their mathematical ideas, connecting problems with pictures or diagrams and mathematical expressions. They can make mathematical models appropriately based on contextual issues given by students with mathematical styles. Impulsive cognitive skills are still not right in expressing mathematical ideas, connecting problems with pictures or diagrams, and mathematical expressions. They can make mathematical models appropriately based on given contextual problems. It can be seen from the answers the information that should have been included in the completion was lost. Students with better initial abilities will have better communication skills because they have more provisions to support and assist them in working on mathematical communication skills test questions.

**DAFTAR PUSTAKA**


pembelajaran matematika di sekolah dasar. EduHumaniora Jurnal Pendidikan Dasar Kampus Cibiru, 6(2).


