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Pseudo-Evaluative Behavior in Mathematical Problem Solving: An Analysis of Students' Metacognitive Processes in Matrix Addition

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Abstract
Metacognitive regulation is widely acknowledged as a key component that supports students in managing their thinking during mathematical problem solving. This study investigates the metacognitive processes demonstrated by a student while completing matrix addition tasks, with a specific focus on the components of planning, monitoring, and evaluating. The research was conducted in a public senior high school in North Halmahera during November and December 2025. One eleventh grade student was purposively selected to provide an in-depth illustration of metacognitive regulation. A qualitative case study design was employed, and data was gathered through written work, direct observation, and semi structured interviews. Analysis was carried out using metacognitive process indicators to capture how the participant organized, controlled, and reviewed each stage of problem solving. The findings show that the participant performed planning effectively by identifying relevant information and determining an approach to the task, and monitoring was evident throughout the solution process. The evaluating phase appeared limited because the participant mainly checked the final answer without engaging in deeper reflection such as analyzing sources of error or comparing alternative procedures. This pattern is characterized in the study as pseudo evaluative behavior, indicating evaluation that is present yet incomplete. The study underscores the need for instructional practices that intentionally strengthen student's evaluative metacognition during mathematical problem solving.

Keywords: evaluating; matrix addition; metacognitive regulation; monitoring; planning; pseudo-evaluative behavior; qualitative case study

1. INTRODUCTION

Metacognition, which refers to the ability to be aware of, manage, and evaluate one's own thinking processes, has long been recognized as a critical element in learning. It encompasses not only knowledge about what is being learned but also how learning strategies are planned, implemented, and refined independently. Research indicates that metacognition contributes significantly to the quality of learning by enabling students to plan, monitor, and evaluate the strategies they use in completing academic tasks (Theobald, 2021). Through these functions, metacognition supports students in becoming self-regulated learners and strengthens higher order thinking skills such as problem solving and decision making.

In the context of mathematics education, particularly linear algebra, metacognition holds a strategic position because the content is often perceived as abstract and challenging. Many students rely on memorizing procedures without understanding the underlying mathematical meaning, especially in topics that require symbolic representation and logically structured reasoning (Jupri & Sispiyati, 2020). Matrix addition is one such topic in which conceptual misunderstandings frequently arise, as students must recognize dimension compatibility, structural properties, and operational rules. Metacognitive skills allow students to evaluate their solution steps, reinterpret mistakes, and improve their understanding through reflective thinking (Güner & Erbay, 2021).

Previous studies on metacognition and mathematics have primarily focused on the relationship between metacognitive levels and academic performance or on the assessment of metacognitive ability through scales or surveys (Muncer et al., 2022). Studies that examine how metacognitive processes operate in concrete mathematical problem-solving situations are still limited, particularly those that analyze students thinking directly through reflective activity during task completion. This creates a need for research that provides deeper insight into how metacognitive processes emerge and develop as students work through specific mathematical problems. Such an approach has the potential to illuminate learning as a dynamic and internal process rather than one that is assessed solely through final products or outcomes (Sutama et al., 2021). Such a shift in focus is crucial for identifying subtle patterns of reasoning, including incomplete or superficial forms of evaluation that may appear adequate but do not support meaningful problem solving.

Building on this need for a more process oriented understanding of metacognition. Flavell (1979) notes that metacognition involves not only awareness of one's thinking processes but also the ability to control, evaluate, and adjust strategies during learning. This theoretical perspective underscores that metacognition is inherently regulatory, involving deliberate decisions made throughout problem solving rather than at isolated points. When applied to mathematics learning, this view suggests that evaluating one's reasoning requires more than simply checking whether an answer is correct. It involves examining the logic behind each step, questioning the appropriateness of procedures, and considering alternative approaches. Emphasizing these regulatory components provides a foundation for investigating behaviors that resemble evaluation yet lack depth, which is central to understanding pseudo evaluative tendencies in mathematical contexts.

In response to these theoretical and empirical needs, the present study offers contributions at both conceptual and practical levels. Theoretically, this study contributes to understanding metacognition as an observable and analyzable process in mathematics learning, particularly in topics that require formal symbolic representation. This aligns with the view of Hidayat, Hermandra, & Ying (2023) that metacognition involves monitoring, cognitive control, and evaluation of strategies in problem solving. Practically, the findings can assist educators in designing learning environments that promote self-reflection, the monitoring of thinking processes, and the evaluation of mathematical problem-solving strategies. Through this lens, the study enriches research on metacognition in mathematics education by offering a more focused understanding of the internal cognitive dynamics that unfold during mathematical task completion, which is essential for analyzing the emergence of pseudo evaluative behavior in problem solving.

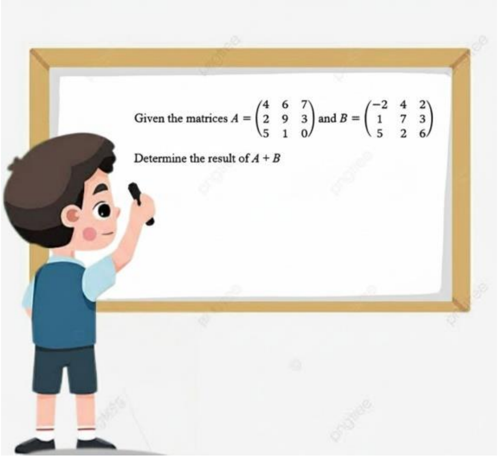
2. RESEARCH METHOD

This study employed a qualitative approach with a case study design. This approach was selected because the research sought to gain an in depth understanding of the student's metacognitive thinking processes while completing matrix addition tasks in a mathematics learning context. A case study design made it possible to explore the phenomenon in detail within its real setting, allowing the student's thinking processes to be observed and analyzed directly as they occurred naturally during learning activities. This methodological choice aligns with the view of Poth et al. (2021) who explain that case studies are particularly suitable when the objective of a study is to explore a specific phenomenon comprehensively within a bounded context.

The study involved a single student as the research participant. The participant was selected through purposive sampling based on specific criteria relevant to the aims of the research. The student was chosen because they possessed sufficient mathematical ability and were able to reflect on their thinking while solving mathematical problems. The inclusion of only one participant is appropriate in qualitative inquiry when the intention is not to generalize findings but to achieve depth of analysis regarding the cognitive processes and learning experiences of an individual.

The research was conducted at a public senior high school in North Halmahera as part of the formal mathematics learning environment at the upper secondary level. Data collection took place from November to December 2025. This time frame was selected based on the alignment of the school's instructional schedule and the researcher's access to the participant.

To collect comprehensive data on the student's metacognitive thinking, the study utilized three complementary instruments. The first instrument was a matrix addition task designed to elicit the student's reasoning processes while working through the mathematical problem. The second instrument was an observation sheet that documented the student's actions, strategies, expressions, and spontaneous responses during problem solving. Through these observations, indicators of metacognition such as monitoring, evaluating, and self correction could be identified. The third instrument consisted of a semi structured interview protocol administered after the completion of the task. The interview was conducted to uncover the student's internal reasoning, justification for strategy choices, and reflections on errors made during the problem-solving process. The combination of these instruments was selected to capture the phenomenon from multiple perspectives and contexts, thereby enhancing the credibility of the data through methodological triangulation (Santos et al., 2020). Furthermore, the use of semi structured interviews offered flexibility in probing the participant's responses while allowing the exploration of ideas in depth (DeJonckheere & Vaughn, 2019). The matrix addition task used in the study is presented in Figure 1.



Given the matrices $A = \begin{pmatrix} 4 & 6 & 7 \\ 2 & 9 & 3 \\ 5 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 4 & 2 \\ 1 & 7 & 3 \\ 5 & 2 & 6 \end{pmatrix}$

Determine the result of $A + B$

Figure 1. The Matrix Addition Task

The data analysis process in this study followed a qualitative descriptive procedure that aimed to identify and interpret the student's metacognitive activities while solving matrix addition tasks. Consistent with the interactive analysis model described by Miles, Huberman & Saldaña (2014), the entire process was conducted through interconnected stages that unfolded continuously during and after data collection. The analysis began with the transcription of verbal expressions produced during the task, the organization of written work, and the compilation of field notes derived from direct classroom observation. These sources were examined collectively to form an initial understanding of the student's reasoning and to prepare the data for systematic reduction and interpretation.

The metacognitive framework proposed by Schraw & Moshman (1995) served as the conceptual lens that guided the categorization of the student's thinking processes. During the reduction stage, the researcher selected and condensed relevant segments of data drawn from observations, problem solving records, and interview transcripts. Each segment was coded according to whether it reflected planning activities, monitoring behaviors, or evaluative judgments. This stage helped refine the focus of the analysis and facilitated the identification of meaningful cognitive actions demonstrated by the student. The coded data were then organized into narrative descriptions, analytic tables, and thematic matrices to support the presentation stage. These structured displays allowed the researcher to observe how the student's metacognitive processes unfolded, shifted, and interacted while completing the matrix addition task.

At the final stage, the researcher interpreted the patterns that emerged from the organized data to formulate conclusions about the student's metacognitive engagement. This interpretive process included repeated verification to ensure the trustworthiness of the findings. Triangulation across written documentation, audio recordings, and observational field notes were conducted to confirm the consistency of identified behaviors. Particular attention was directed to moments in which the student expressed superficial evaluative statements that resembled genuine evaluation but were not supported by substantive reasoning. Through the integrated procedures of data transcription, reduction, presentation, verification, and interpretive synthesis, the analysis produced a detailed and reliable account of the metacognitive processes displayed during the completion of the matrix addition task.

To provide a clearer picture of how the analytical decisions were made and how each segment of data was classified, the coding scheme used in this study is presented in table 1. This table summarizes the metacognitive categories, their operational definitions, the behavioral indicators used during the reduction process, and the primary data sources from which each code was derived. It serves as an analytic map that connects the theoretical framework to the empirical evidence gathered throughout the task.

Table 1. Coding Categories, Definitions, and Evidence Sources

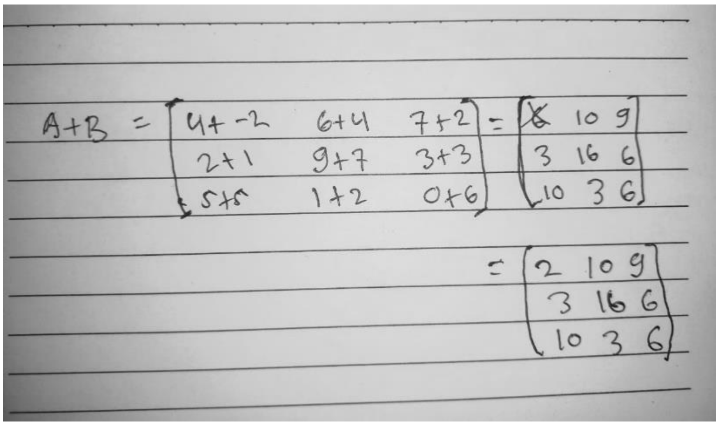
Code	Category	Operational Definition	Indicators / Behavioral Evidence	Primary Data Sources
PL	Planning	Student's initial efforts to understand the problem, set goals, and determine strategies before executing the task.	Rereading the problem; examining the structure of both matrices; verbalizing strategy such as *"I will add the corresponding elements."*	Observation notes; written work; task-based verbal expressions
MN	Monitoring	Ongoing self-checking processes that occur while performing each computation step.	Pauses during calculation; rechecking element-wise results; visible revisions on written work (e.g., reviewing *4 + (–2)*).	Observation notes; written traces on task sheet
EV	Evaluation	Genuine assessment of the correctness and completeness of the final answer after completing the task.	Reflecting on whether the result "makes sense"; comparing the final matrix with expected addition rules; providing justification for the correctness of the answer.	Post-task interview; verbal reflections; final written answer
PE	Pseudo-Evaluation	Superficial evaluative statements resembling evaluation but lacking verification,	Declaring *"I think it is correct"* without checking operations; giving a confirmation statement	Interview transcript; observation notes

	justification, or cognitive review.	without evidence; absence of analytical reasoning.

3. RESULT AND DISCUSSION

At the early stage of solving the task, the student demonstrated planning ability by rereading the problem and examining the structure of both matrices before performing the addition. Observational notes indicate that the student explicitly articulated the strategy to be used by stating, “I add the corresponding elements,” which reflects an initial awareness of the procedural requirements of matrix addition. This behavior shows that the student understood the foundational rule governing the operation and was able to establish a sequence of actions before writing the solution. Such activity aligns with the planning component described by Schraw & Moshman (1995), which emphasizes the capacity to set strategies and define goals prior to engaging in a cognitive task. This initial planning stage provided the groundwork for the student’s subsequent actions and illustrated how preparatory thinking supports the structuring of problem-solving processes in mathematics.

During the execution of the task, the student also exhibited monitoring behavior as evidenced by several pauses and self-checks that occurred while writing the solution. These moments of reflection were captured through direct observation and were further confirmed by physical traces on the written work, such as erasures and overwritten numbers. A notable instance occurred in the first row and first column, where the student initially computed the element 4 plus negative 2 and recorded the result as 6, as shown in the student’s work presented in Figure 2. The student then paused, reviewed the computation, and subsequently revised the answer. This sequence illustrates the student's attempt to regulate their thinking by assessing the correctness of each intermediate step, demonstrating that monitoring was occurring as an active component of their metacognitive engagement.



$$A+B = \begin{bmatrix} 4+-2 & 6+4 & 7+2 \\ 2+1 & 9+7 & 3+3 \\ 5+5 & 1+2 & 0+6 \end{bmatrix} = \begin{bmatrix} \cancel{6} & 10 & 9 \\ 3 & 16 & 6 \\ 10 & 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 10 & 9 \\ 3 & 16 & 6 \\ 10 & 3 & 6 \end{bmatrix}$$

Figure 2. The student’s work

The student subsequently crossed out the incorrect result and replaced it with the correct value, which was 2. This act of revising the answer reflects a form of self-correction that emerges from self-monitoring, demonstrating the student's recognition of an operational error and the attempt to repair it. Such behavior corresponds to the metacognitive characteristics described in earlier work. However, the monitoring displayed by the student appeared to be largely reactive, triggered only when an error became visibly apparent, rather than a proactive and systematic review of the entire sequence of problem-solving steps. This pattern suggests that the student's regulation of thinking was inconsistent, occurring only at isolated points of conflict rather than through an ongoing evaluative process.

As the task progressed, the evaluation phase revealed further complexities in the student's metacognitive functioning. Although the student conducted a final check after completing the written solution, a deeper analysis of the interview data and written work indicates that this evaluation did not align with the characteristics of mature metacognitive evaluation. Instead, it resembled a pseudo evaluative process. The student explained the reasoning by stating, "I corrected the part that was wrong after realizing the negative sign had not been counted. After that I checked again until I felt sure the result was correct." At first glance, this statement appears to represent purposeful reflection. However, closer examination shows that the student was unable to provide a systematic justification for how the correctness of the answer was determined. When asked to explain the mechanism used to ensure accuracy, the student responded, "Usually if the final result looks right and nothing seems off, then it is correct." This response illustrates a fundamental limitation in the evaluative process, revealing a reliance on subjective impressions rather than verifiable mathematical reasoning.

A central issue that emerged was that the student's evaluation was not grounded in evidence but instead relied on perceptions of plausibility. Rather than verifying correctness through stepwise checking or objective mathematical justification, the student tended to judge accuracy based on whether the answer appeared reasonable. Such plausibility based evaluation contradicts the ideal characteristics of metacognitive evaluation, which emphasize accuracy of self-judgment grounded in objective evidence and conceptual verification, as described by Craig et al. (2020). No explicit effort was observed to align the obtained results with the rules governing matrix addition or to verify the structural consistency of the resulting matrix.

Additionally, although the student corrected the earlier computational error, the reasoning provided did not demonstrate an understanding of why the error occurred. The explanation "the negative sign was not counted" did not reflect an awareness of deeper contributing factors, such as insufficient attention to sign conventions or difficulty in aligning corresponding elements within matrices. This inability to articulate the source of the error suggests that the evaluative process did not extend into reflective analysis of the underlying thinking mechanisms. This phenomenon mirrors the findings of Kruger & Dunning (1999), who noted that individuals with limited competence often lack the capacity to accurately detect flaws in their cognitive processes, resulting in self-evaluation that is imprecise and incomplete.

Furthermore, the student's evaluation was oriented exclusively toward the correctness of the final answer rather than toward the quality or validity of the process leading to it. When asked whether alternative strategies or a more systematic checking procedure might be used, the student replied, "No, I just see whether the final result is correct or not." This response suggests that the student's evaluative activity remained confined to result checking, with no attention to reviewing the steps taken, assessing the effectiveness of strategies, or formulating improvements for future problem solving. The absence of process oriented reflection indicates that the student did not engage in the deeper form of metacognitive evaluation described by Çini et al. (2023) which requires examining the appropriateness of strategies and considering refinements for subsequent tasks. In this sense, the evaluation demonstrated by the student lacked depth and did not reflect the systematic procedural analysis associated with mature metacognitive engagement.

Taken together, these findings illustrate that the student was able to demonstrate planning and monitoring at a basic level, particularly through the initial selection of a strategy and the detection of operational errors during the task. These forms of engagement indicate that the student has begun to develop foundational aspects of metacognition. However, the evaluation component emerged as the weakest aspect of the student's metacognitive profile. The evaluation conducted did not reflect deep analysis of the procedures used, nor did it show evidence based self-judgment or reflective awareness of strategic limitations. Rather than evaluating the reasoning process, the student focused predominantly on the correctness of the result and did not demonstrate reflection on strategy effectiveness or consideration of improvements for future tasks.

This pattern aligns with the findings of Scheibe et al. (2023) who observed that students frequently fall into superficial forms of evaluation when their metacognitive structures are not yet fully developed. Consequently, the evaluative behavior observed in this study can be classified as pseudo evaluative, a surface level form of evaluation that appears legitimate but lacks genuine reflective engagement with the steps, strategies, and underlying mathematical concepts involved in solving the task. Such evaluation does not reflect deeper reasoning or systematic procedural analysis and therefore cannot be categorized as mature metacognitive evaluation within the theoretical framework.

Table 2. Summary of Findings

Metacognitive Aspect	Key Findings	Student Evidence	Coding Category	Interpretation	Pseudo-Evaluative Indicator
Planning	The student reread the problem, examined both matrices, and articulated the strategy before solving.	"I add the corresponding elements."	PL1 Identifying task requirements • PL2 Selecting strategy	Demonstrates awareness of the task structure and ability to set a strategic plan.	Not applicable
Monitoring	The student paused to recheck local computations and corrected visible errors during the process.	Correction of 4 + (–2) from 6 to 2; observable pauses and revisions.	MO1 Local checking • MO2 Error detection • MO3 Reactive monitoring	Monitoring occurred only when errors became obvious, not systematically across all steps.	Not applicable
Evaluating	Evaluation focused solely on checking the final answer without reviewing procedures or verifying correctness systematically.	"If the final result looks right, it means it is correct."	EV1 Result checking only • EV2 Plausibility judgment • EV3 No procedural review	Reveals shallow evaluation lacking step-by-step verification or conceptual justification.	**Yes – Pseudo-Evaluative Behavior (EV4)**
Error Identification	The student was unable to specify the source of errors and provided nonspecific explanations.	"The minus was not counted."	ER1 Unspecific error attribution	Indicates limited awareness of why the error occurred and how it developed.	**Contributes to pseudo-evaluation**
Strategic Reflection	No effort was made to assess the effectiveness of strategies or to plan improvements for future tasks.	"No, I just see whether the result is correct or not."	RS1 No strategy evaluation • RS2 No improvement planning	Shows evaluation centered entirely on the final result, not on the reasoning or process.	**Yes – Lack of reflective evaluation supports pseudo-evaluation**

4. CONCLUSION

The findings of this study reveal that the student demonstrated emerging metacognitive engagement, particularly in the domains of planning and monitoring, as shown through the selection of an initial strategy and the ability to detect and correct operational errors during the matrix addition task. However, the evaluative component of metacognition appeared considerably weaker and was characterized by pseudo-evaluative judgments that lacked systematic justification and evidence-based reasoning. This imbalance suggests that while foundational metacognitive processes have begun to develop, the student has not yet reached a mature level of reflective evaluation that integrates strategic awareness, verification procedures, and conceptual grounding. Overall, the study underscores the necessity of instructional designs that explicitly cultivate deeper evaluative thinking to support the development of comprehensive metacognitive competence in mathematical problem solving.

5. RECOMMENDATION

Based on the patterns identified in the student's metacognitive engagement, this study recommends that future instructional practices strengthen students' evaluative metacognitive skills through explicit scaffolding and structured opportunities for reflection. Teachers can provide guided self-evaluation prompts, invite students to conduct error analysis, and model verbal reasoning that demonstrates how to justify mathematical solutions using clear evidence rather than relying on surface level impressions. Embedding metacognitive checklists within problem solving activities may also help students internalize more systematic ways of verifying their work and recognizing how their strategies align with underlying mathematical concepts. Furthermore, future research with larger and more diverse groups of students is encouraged to deepen understanding of how superficial forms of evaluation develop and how targeted instructional support can foster more authentic evaluative reasoning. Through these efforts, mathematics learning can better promote comprehensive metacognitive growth.

6. REFERENCE

Çini, A., Järvelä, S., Dindar, M., & Malmberg, J. (2023). How multiple levels of metacognitive awareness operate in collaborative problem solving. *Metacognition and Learning*, *18*(3), 891–922. https://doi.org/10.1007/S11409-023-09358-7

Craig, K., Hale, D., Grainger, C., & Stewart, M. E. (2020). Evaluating metacognitive self-reports: systematic reviews of the value of self-report in metacognitive research. *Metacognition and Learning*, *15*(2), 155–213. https://doi.org/10.1007/S11409-020-09222-Y

DeJonckheere, M., & Vaughn, L. M. (2019). Semistructured interviewing in primary care research: A balance of relationship and rigour. *Family Medicine and Community Health*, *7*(2). https://doi.org/10.1136/FMCH-2018-000057

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist*, *34*(10), 906–911.

https://doi.org/10.1037/0003-066X.34.10.906

Güner, P., & Erbay, H. N. (2021). Metacognitive Skills and Problem-Solving. *International Journal of Research in Education and Science*, *7*(3), 715–734. https://doi.org/10.46328/IJRES.1594

Hidayat, R., Hermandra, & Ying, S. T. D. (2023). The sub-dimensions of metacognition and their influence on modeling competency. *Humanities and Social Sciences Communications*, *10*(1). https://doi.org/10.1057/S41599-023-02290-W

Jupri, A., & Sispiyati, R. (2020). Students' algebraic proficiency from the perspective of symbol sense. *Indonesian Journal of Science and Technology*, *5*(1), 86–94. https://doi.org/10.17509/IJOST.V5I1.23102

Kruger, J., & Dunning, D. (1999). Unskilled and unaware of it: How difficulties in recognizing one's own incompetence lead to inflated self-assessments. *Journal of Personality and Social Psychology*, *77*(6), 1121–1134. https://doi.org/10.1037/0022-3514.77.6.1121

Miles, Matthew B.; Huberman, A. Michael; Saldaña, J. (2014). Qualitative Data Analysis: A Methods Sourcebook. In *SAGE Publications, Inc.* (3rd ed.). SAGE Publications, Inc.

Muncer, G., Higham, P. A., Gosling, C. J., Cortese, S., Wood-Downie, H., & Hadwin, J. A. (2022). A Meta-Analysis Investigating the Association Between Metacognition and Math Performance in Adolescence. *Educational Psychology Review*, *34*(1), 301–334. https://doi.org/10.1007/S10648-021-09620-X

Poth, C. N., Bulut, O., Aquilina, A. M., & Otto, S. J. G. (2021). Using Data Mining for Rapid Complex Case Study Descriptions: Example of Public Health Briefings During the Onset of the COVID-19 Pandemic. *Journal of Mixed Methods Research*, *15*(3), 348–373. https://doi.org/10.1177/15586898211013925

Santos, K. da S., Ribeiro, M. C., de Queiroga, D. E. U., da Silva, I. A. P., & Ferreira, S. M. S. (2020). The use of multiple triangulations as a validation strategy in a qualitative study. *Ciencia e Saude Coletiva*, *25*(2), 655–664. https://doi.org/10.1590/1413-81232020252.12302018

Scheibe, D. A., Was, C. A., Dunlosky, J., & Thompson, C. A. (2023). Metacognitive Cues, Working Memory, and Math Anxiety: The Regulated Attention in Mathematical Problem Solving (RAMPS) Framework. *Journal of Intelligence*, *11*(6). https://doi.org/10.3390/JINTELLIGENCE11060117

Schraw, G., & Moshman, D. (1995). Metacognitive theories. *Educational Psychology Review*, *7*(4), 351–371. https://doi.org/10.1007/BF02212307/METRICS

Sutama, S., Anif, S., Prayitno, H. J., Narimo, S., Fuadi, D., Sari, D. P., & Adnan, M. (2021). Metacognition of Junior High School Students in Mathematics Problem Solving Based on Cognitive Style. *Asian Journal of University Education*, *17*(1), 134–144. https://doi.org/10.24191/AJUE.V17I1.12604

Theobald, M. (2021). Self-regulated learning training programs enhance university students' academic performance, self-regulated learning strategies, and motivation: A meta-analysis. *Contemporary Educational Psychology*, *66*. https://doi.org/10.1016/J.CEDPSYCH.2021.101976