



APOS Theory and Relational Understanding: A Systematic Literature Review

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Abstract

Relational understanding is an essential skill that reflects students' mastery of the relationships between mathematical concepts, while APOS Theory provides a framework for analyzing the mental construction of this understanding. This study aims to identify and compare the application of APOS Theory and relational understanding in the context of mathematics learning. This study used the Systematic Literature Review (SLR) method based on the PRISMA framework. The literature search was conducted in the Scopus and Google Scholar databases with publications limited to 2014 to 2024, focused on elementary and secondary school levels. Of the 228 identified studies, 11 empirical articles were selected for analysis. The review results showed significant differences in the focus of the material; APOS Theory is predominantly applied to number topics and is often used as a basis for developing assessment instruments. In contrast, relational understanding is more widely studied in algebra topics. Furthermore, it was found that the problem-solving approach is a major factor influencing the formation of students' relational understanding. These findings offer educators insights into selecting pedagogical approaches that are suitable for the material's characteristics.

Keywords: APOS Theory; Conceptual Understanding; Relational Understanding

Abstrak

Pemahaman relasional merupakan kemampuan esensial yang mencerminkan penguasaan siswa terhadap hubungan antar konsep matematika, sementara Teori APOS menyediakan kerangka kerja untuk menganalisis konstruksi mental pemahaman tersebut. Penelitian ini bertujuan untuk mengidentifikasi dan membandingkan penerapan Teori APOS serta pemahaman relasional dalam konteks pembelajaran matematika. Penelitian ini menggunakan metode Systematic Literature Review (SLR) dengan kerangka kerja PRISMA. Penelusuran literatur dilakukan pada basis data Scopus dan Google Scholar dengan batasan publikasi tahun 2014 hingga 2024, difokuskan pada jenjang sekolah dasar dan menengah. Dari 228 studi yang teridentifikasi, sebanyak 11 artikel empiris dipilih untuk dianalisis. Hasil tinjauan menunjukkan perbedaan signifikan dalam fokus materi; Teori APOS lebih dominan diterapkan pada topik bilangan dan sering digunakan sebagai basis pengembangan instrumen penilaian. Sebaliknya, pemahaman relasional lebih banyak dikaji dalam topik aljabar. Selain itu, ditemukan bahwa pendekatan pemecahan masalah menjadi faktor utama yang memengaruhi terbentuknya pemahaman relasional siswa. Temuan ini memberikan wawasan bagi pendidik dalam memilih pendekatan pedagogis yang sesuai dengan karakteristik materi.

Kata Kunci: Pemahaman Konsep; Pemahaman Relasional; Teori APOS

1. NTRODUCTION

Mathematics is the foundation of the educational curriculum, not only for academic success but also for developing logical reasoning and analytical skills needed in life. According to James and James (Aspia, 2021), mathematics is a science related to logic, recognizing shapes, the arrangement of quantities, and other relational concepts, which are numerous and divided into three fields: algebra, analysis, and geometry. Given this crucial role, the primary expectation in mathematics education is to equip students with a solid understanding of mathematical concepts. However, the reality on the ground often contradicts this expectation. A phenomenon that occurs and is a concern in mathematics classes is the tendency for learning to focus on memorization. Hasanah (2020) argues that mathematics inherently demands a deep understanding, rather than simply the ability to memorize procedures or algorithms. However, many students mistakenly view mathematics as simply a collection of random rules that must be memorized. In fact, mathematics should be understood as a system in which each concept is interconnected. This reliance on rote learning leads to what Skemp (1976) calls "instrumental understanding," a shallow understanding in which students can execute formulas to solve routine problems but remain unaware of the underlying logic or "why" behind their actions.

The impact of this shallow understanding cannot be underestimated. When students rely solely on instrumental understanding, they often encounter impasses when faced with non-routine problems. Rokhmawati and Rahayu (2023) highlight that while procedural memory can help students survive standardized tests, it fails to provide the flexibility needed to apply knowledge in new contexts. This lack of depth becomes a major obstacle to achieving long-term learning goals (Syaifar et al., 2022), as students struggle to see the connections between the concepts they have learned. Consequently, there is an urgent need to shift the pedagogical focus from simply getting the right answer to cultivating relational understanding. Ideally, students should be able to integrate separate mathematical concepts into a coherent understanding, enabling them to explain their reasoning and apply concepts flexibly (Mefiana & Juandi, 2023). However, achieving this level of understanding is not an easy task; a structured theoretical perspective is needed to analyze how students construct mathematical meaning in their minds.

To bridge the gap between rote learning and deep conceptual mastery, educators need a robust framework for diagnosing and facilitating cognitive development. This is where APOS (Action, Process, Object, and Schema) Theory becomes crucial. Based on Piaget's constructivist principles and reflective abstraction, APOS Theory offers a detailed map of the mental steps students take to acquire new concepts (Cetin & Dubinsky, 2017). This theory states that understanding is a process that develops over time, not simply understanding or not understanding. It begins with Action, where students transform physical or mental objects using clear, step-by-step algorithms (Tatira, 2021) Through practice and reflection, these actions are internalized into a Process, allowing students to

perform operations mentally without external assistance. As students master them, these processes are packaged into an Object, a static entity that can be manipulated and acted upon. Eventually, these actions, processes, and objects are organized into a Schema, which represents a deep relational understanding of the topic.

By applying APOS Theory, educators gain a powerful diagnostic tool. Rather than simply labeling a student as "weak," a teacher can identify that a student is stuck at the Action stage while their peers have moved to the Process stage. Anam (2021) suggests that this theory provides a clear picture of how mathematical knowledge is formed, allowing teachers to design interventions tailored to students' specific cognitive levels. Furthermore, Fransisca et al. (2021) emphasize that guiding students through these stages ensures a more comprehensive mastery of science and technology, which is essential for modern life. In essence, APOS Theory provides an understanding of the "how" to the "what" of relational understanding. This is in line with the constructivist view which encourages students to think broadly and apply the theories they learn to everyday problems (Suparlan, 2019). Therefore, it is hoped that mathematics learning is not only about passing exams, but also about building a sustainable intellectual framework.

Despite the clear relevance between APOS Theory and relational understanding, existing research often addresses these two constructs separately. A literature review reveals fragmentation. For example, (Syafri, 2016) conducted a literature review focusing on APOS Theory to analyze cognitive structures. This study effectively highlighted students' internal mental processes, but did not deeply connect these mechanisms with indicators of relational understanding in specific mathematical topics. In contrast, Mefiana and Juandi (2023) conducted a systematic review of relational understanding in the context of problem-solving, but their research focused more on the outcomes of understanding rather than the developmental stages (Action to Schema) that precede it. Similarly, Sudrajat (2022) compared relational and instrumental understanding but did not integrate the APOS framework as a tool to analyze the process of achieving this understanding.

This fragmentation highlights a gap in the current literature. Currently, there is a lack of comprehensive studies that specifically link the application of APOS Theory and relational understanding. Furthermore, existing reviews have not sufficiently distinguished how these theories are applied differently to different mathematical topics. It remains unclear, for example, whether APOS Theory is more dominantly applied in algebra compared to geometry, or how indicators of relational understanding manifest differently in number theory compared to calculus. Understanding the specific nuances of these topics is crucial because the nature of mathematical content significantly influences the cognitive demands placed on students. Furthermore, the specific factors influencing relational understanding within the APOS stages framework have not been

comprehensively mapped. Without this unified view, educators and researchers will lack a complete picture of how best to facilitate student learning, particularly in bridging the transition from early stages to deep mastery of the material.

This study aims to address this gap by conducting a Systematic Literature Review (SLR) that integrates APOS Theory with relational understanding. The novelty of this study lies in its specific focus on identifying how the APOS approach is adapted to various mathematical topics to foster relational understanding, and in mapping the determinants of student success. In contrast to previous research that tends to only look at one aspect, this study presents a comprehensive synthesis of teaching strategies that bridge students' mental processes with in-depth conceptual understanding. Therefore, based on the theoretical considerations and practical needs outlined previously, specifically, this study aims to: (1) identify the dominant mathematical topics studied in APOS Theory and relational understanding; (2) analyze how APOS Theory is applied, both in the mathematics learning process and in determining student abilities; and (3) reveal what factors influence the formation of students' relational understanding. By achieving these objectives, this study seeks to provide an applicable guide for educators in determining learning strategies that are not only theoretically sound, but also relevant to each mathematical topic.

2. RESEARCH METHOD

This research uses the systematic literature review (SLR) method. The process carried out in systematic literature review research is to identify, evaluate, and analyze certain topics systematically and comprehensively, with the aim of providing a clear and comprehensive picture of the research conducted on the topic (Febriatama, 2023). The research protocol used is based on the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analysis) framework. The entire process from the identification stage to the selection stage of the articles to be reviewed is presented in the flowchart in Figure 1.

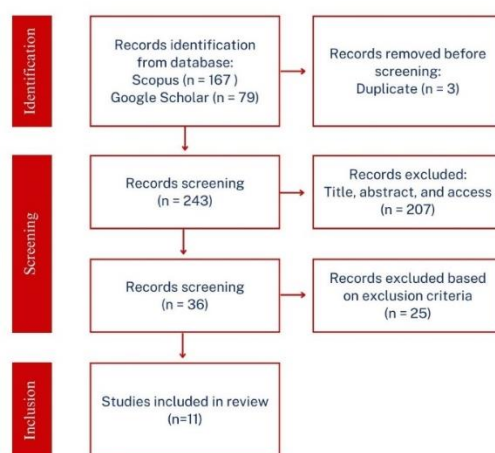


Figure 1. Research Flow

The works to be reviewed were identified using the keywords APOS and Relational Understanding / APOS AND “Relational Understanding” limited to the period from 2014 to 2024, with data sources from Scopus and Google Scholar. In the Scopus data source, 167 works were found, and in the Google Scholar data source, 79 works were found, but 2 of them were excluded as they were considered duplicate articles. Next, the process of screening articles relevant to the research topic was conducted based on the inclusion and exclusion criteria presented in Table 1.

Table 1. Inclusion and Exclusion

Inclusion	Exclusion
Article discussing the APOS theory or relational understanding	The research does not discuss the APOS theory or relational understanding
Articles with qualitative and quantitative research	Articles with a literature review method
Articles published between 2014 and 2024	Articles published before 2014 and after 2024
Articles that have been published (released) with sources from Scopus and Google Scholar	Research that has been published outside the Scopus and Google Scholar database
Available in full text and has undergone the peer-review process and has been published	Articles available only in abstract form and not through the peer-review process
Research subjects in the article from the educational levels of elementary and middle schools	Research subjects outside the education levels of elementary and middle schools
The article is written in Indonesian and English	Articles written in languages other than Indonesian and English

In this process, 37 articles were obtained, all of which are scientific journal articles. Next, in the third stage, the articles are read and identified, resulting in 25 articles that do not discuss relational understanding and the APOS theory for elementary to high school students, which are not accessible, and are systematic literature review studies. From that stage, there are 12 articles that will be reviewed. Data from eligible articles were extracted and entered into a database, which included author identity, year of publication, and research results. The extracted data were then analyzed and synthesized to answer the research questions.

3. RESULT AND DISCUSSION

3.1 Result

Data Analysis

Data from eligible articles were extracted and entered into a database, which included author identity, year of publication, and research results. Data analysis in this study was conducted through three main stages: data reduction, data presentation, and conclusion drawing. First, data from 11 articles that met the inclusion criteria were extracted into a database matrix that included author identity, year of publication, research design, and key findings. Second,

content analysis was conducted to group and synthesize the data to achieve the stated research objectives. Quantitative data, including the frequency of occurrence of mathematical topics (Algebra, Number, Geometry) and the application of APOS theory, were analyzed using simple descriptive statistics (percentages) and presented in pie charts and bar charts. Meanwhile, qualitative data regarding factors influencing relational understanding were analyzed using narrative synthesis techniques to identify patterns, similarities, and differences across studies. Finally, all findings were synthesized to answer the research objective regarding the relationship between APOS Theory and relational understanding.

Topics applied to the APOS theory approach, from 6 articles reviewed, 3 papers were found discussing the topic of numbers, 1 paper discussing the topic of algebra, and 2 papers discussing the topic of geometry, with the percentage between topics presented in Figure 2. From Figure 2, it can be seen that the most studied topics in the APOS theory approach are topics that discuss numbers, while very little research discusses algebraic topics in research related to APOS theory.

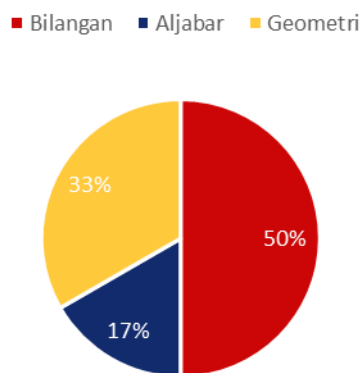


Figure 2. Diagram of APOS Theory Material Topics

Papers discussing relational understanding from the 5 articles reviewed, 1 paper was found discussing the topic of numbers, 3 papers discussing the topic of algebra, and 1 paper discussing the topic of geometry, with the percentage distribution of topics presented in Figure 3. From Figure 3, it is known that the topic of Algebra dominates research related to relational understanding at the elementary and secondary school levels.

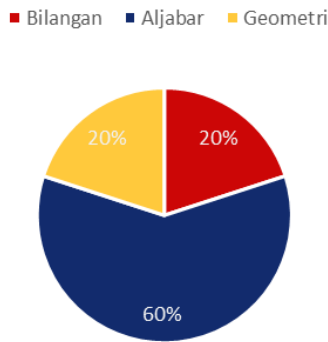


Figure 3. Diagram of Relational Understanding Topic Material

The results of the study from 6 articles discussing the topic of APOS theory show that the use of APOS theory is applied to conceptual understanding questions, as a learning model, assignment questions, mental construction test questions, and is used to assess students' level of understanding. Figure 4 presents the number of applications of the APOS theory approach in the research studied. From Figure 4, it can be seen that the APOS theory approach is widely applied as a test instrument.

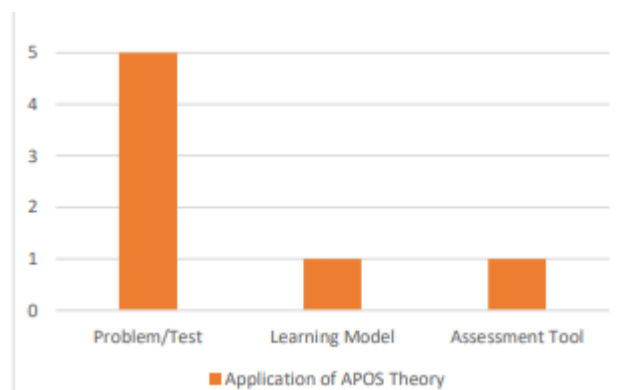


Figure 4. Diagram of the APOS Theory Implementation Instrument

The application of APOS theory in the reviewed articles shows variations in its use to determine students' abilities, such as problem-solving abilities, comprehension abilities, and students' mental constructions, which are presented in Figure 5. Figure 5 shows that the application of APOS theory is widely applied to determine the comprehension abilities of elementary and secondary school students.

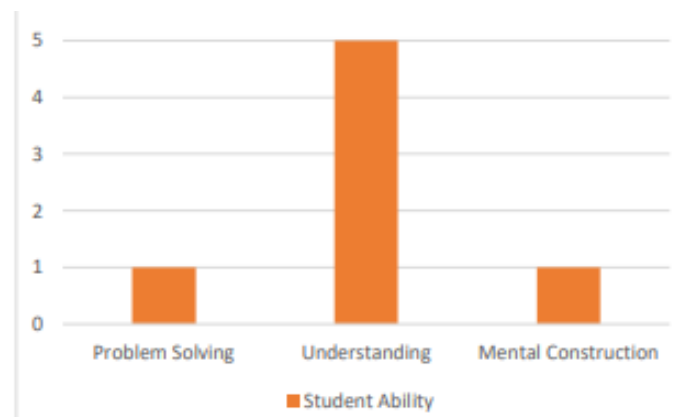


Figure 5. Diagram of the Use of APOS Theory to Identify Students' Ability

Factors that influence students' relational understanding based on the results of the 5-paper study have been identified, including several factors such as gender differences, problem-solving stages, how students express definitions and use strategies, students' understanding models, and how students answer the questions given. These factors are categorized into three main factors: gender differences, understanding models, and problem solving, as presented in Figure 6. In Figure 6, it can be seen that the factor that most influences relational understanding is the way students solve problems.

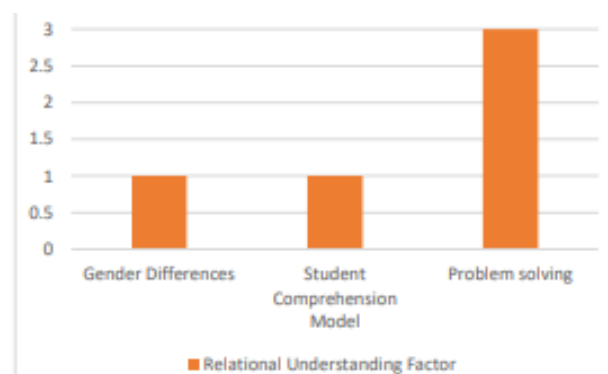


Figure 6. Factors Affecting Students' Relational Understanding

Critical Appraisal

Critical appraisal was conducted on the screened articles using the Preferred Reporting Items for Systematic Literature Review (PRISMA) framework to examine APOS Theory and relational understanding. Each article's eligibility was evaluated based on the inclusion and exclusion criteria in Table 1. 11 articles were selected for analysis. Details of these articles are provided in Tables 2 and 3.

Literature Review Results

From the findings of the literature search, 11 articles were obtained for analysis. The articles are divided into two parts, namely articles discussing the APOS theory and articles discussing

relational understanding. There are 6 articles discussing the APOS theory and 5 articles discussing relational understanding.

Table 2. Literature Research Results Discussing the APOS Theory Topic.

No.	Author	Results
1	Suratih et al., 2023	Students with high initial abilities successfully reach the action, process, object, and schema stages. Students with moderate initial abilities reach the action and process stages, but have not yet reached the object and schema stages. Meanwhile, students with low initial abilities can only reach the action stage and have not yet developed to the process, object, or schema stages.
2	Syam, 2021	The application of the M-APOS learning model in mathematics learning is worth considering as an alternative learning method to improve the mathematical problem solving skills of X grade high school students.
3	Anam, 2021	Of the 2 subjects of different genders, female students can perform all four stages of APOS theory completely and maximally, so they are categorized as having relational understanding. While male students, have not fully performed the stages in APOS theory completely and maximally, especially at the action and object stages, so that the subject is categorized as having an instrumental level of understanding.
4	Bajo-Benito et al., 2021.	Students who use logical relationships in solving problems correctly show better understanding and are categorized as being at the trans-level schema development lever, and students who incorrectly use logical relationships in solving problems are at the inter-level schema development level.
5	Afkhami et al., 2023	There are levels and stages of students in generalizing figurative patterns based on the stages of APOS theory, namely at the level before the action stage and the level before the process/object stage. The level before the action stage consists of emergent action and partial action. While at the level before the process/object stage there are levels of pre-emergent, process/object and partial process/object.
6	Tiengyoo et al., 2024	More than 60% of students at all levels achieved scores above the set criteria, with 80% demonstrating students' understanding of the set

From the 5 articles that discuss relational understanding, the factors that influence students' relational understanding are as follows, gender differences, problem solving stages, how students express definitions and use strategies, students' understanding models, and how students answer the given problems. The research results from the articles reviewed are presented in Table 3.

Table 3. Results of Literature Research that Discusses the Topic of Relational Understanding

No.	Author	Results
1	Rivian & Hidayati, 2024	Female students have higher relational comprehension skills than male students

2	Linda et al., 2024	Problem solving ability affects relational understanding ability. Students who have complete problem solving stages also have complete relational understanding.
3	Andrews & Kaplan, 2020	Large-scale international assessments (ILSAs) such as PISA and TIMSS may not accurately reflect Swedish students' understanding of linear equations.
4	Sumpter & Löwenhielm, 2024	Most students had an operational or relational understanding of equations. However, when the learning outcomes were analyzed more deeply using various methods, it turned out that students' understanding was much more complex than simply categorizing operational and relational understanding
5	Pittalis, 2024	The first group students, showed limited basic understanding of rational numbers, their arithmetic skills and flexibility were also weak. The second group has an adequate basic understanding of rational numbers, but limited arithmetic skills and flexibility, they can complete basic tasks well but struggle with more complex arithmetic problems. The third group excelled in basic tasks and arithmetic, had a strong understanding and flexibility with rational numbers, and could explain their reasoning in arithmetic operations

3.2 Discussion

Mathematical Topics in APOS Theory and Relational Understanding

The APOS theory approach is widely applied to various mathematics topics. In research with elementary to secondary school subjects, APOS theory is widely applied to a number topics. In fact, in its application, APOS theory is used in algebra topics. Dubinsky and McDonald stated that the development and use of APOS theory in various topics including abstract algebra, discrete mathematics, calculus, statistics, and other college-level mathematics subjects. One of the studies that implemented the APOS theory approach to the topic of number material was carried out by Syam (2021) in his research, Student activities within the framework of the APOS theory model modified into the M-APOS learning model that utilizes recitation tasks. The results of the study prove that the ability to understand mathematical concepts of students taught by implementing the M-APOS learning model is higher than students taught without the M-APOS learning model. From these results, it can be shown that the APOS theory with modifications is effectively applied as a mathematics learning model on the topic of numbers.

Relational understanding at the elementary and secondary school levels is widely studied in the topic of algebra. One of them is in a study related to the understanding of 7th-grade students in the material on equations and inequalities conducted by Sumpter and Löwenhielm (2024). The results of the study indicate that majority of students have an operational or relational understanding of equations. However, when the learning

outcomes were analyzed more deeply using various methods, it turned out that students' understanding was much more complex than simply categorizing operational and relational understanding. Students' understanding of equations in mathematics develops gradually and is interconnected between operational and relational understanding. The topic of algebra in mathematics tends to be interesting to study. Because the topic of algebra is one of the topics in mathematics whose discussion focuses on simplification and problem solving by using substitute symbols to represent constants and variables, in this science there are theories such as number theory, geometry, and algebraic theory which are often studied in mathematical research (Sampoerna Academy, 2022).

Application of APOS Theory in Mathematics Learning

Literature analysis on the application of the APOS theory approach at elementary and secondary school levels shows that the APOS theory approach is more widely used in question or test instruments. The stages of APOS theory, namely, action, process, object and scheme are indicators in compiling questions. Supported by research conducted by Suratih et al. (2023) which examined students' conceptual understanding abilities based on APOS theory, in the study, 6 subjects were given a test compiled based on indicators of conceptual understanding and criteria for APOS theory stages. With the conclusion that students who have high initial abilities reach the action, process, object and scheme stages. Students who have moderate initial abilities reach the action and process stages, while the object and scheme stages have not been met. And students with low initial ability categories can only reach the action stage, students in this category have not been able to reach the process, object and scheme stages. In line with the opinion of Cetin and Dubinsky (2017), one of the uses of APOS theory is to provide pedagogical strategies that can guide students in overcoming the difficulties they face when studying various concepts in mathematics. Thus, APOS theory can be used in test instruments to determine students' abilities based on the stages of APOS theory with the aim of helping students' difficulties in learning mathematics.

Application of APOS Theory to Determine Students' Abilities

Through the APOS theory approach, teachers can find out students' understanding and provide appropriate learning. From the study of the article, the use of APOS theory is most often used to determine students' abilities, especially students' comprehension abilities. This understanding ability includes students' conceptual understanding. One of them is in a study conducted by Bajo-Benito et al. (2021) which examined students' understanding of number sequences using APOS theory as a category level in the development of students' mathematical concepts. In this study, the stages of APOS theory used as indicators to categorize the development of students' concepts are the schema stages which are divided into several levels, namely; intra level, inter level, and trans level. The results of the study showed that students who used logical relationships in

solving problems correctly showed better understanding and were categorized as being at the trans level schema development level, and students who incorrectly used logical relationships in solving problems were at the inter level schema development level. The researcher selected the schema stage as a key indicator of student concept development because it represents a network of interconnected structures (including actions, processes, objects, and other schemas) along with the relationships that exist among these structures. Additionally, these way this scheme is conceptualized has been discussed in numerous studies aimed at defining the understanding of different mathematical concepts. This is also in line with the characteristics of the schema stage according to Mulyono (2011), where the schema stage can connect and understand the relationships between actions, processes, objects and other properties that have been understood by an individual. So it can be said that the schema stage in the APOS theory stages can describe a person's understanding based on the relationships from the previous stages, so that if an individual has reached the schema stage, then the previous stages (actions, processes and objects) can also be achieved.

Another study was conducted by Tiengyoo et al. (2024) using the APOS theoretical framework to test students' level of understanding of set material in mathematics learning. In this study, students were categorized into four levels of understanding, namely, action, process, object and scheme, with the results of the study showing that more than 60% of students at all levels scored above the set criteria, with 80% showing students' understanding of sets. Another study that uses APOS theory in giving questions to determine students' understanding of mathematical concepts was conducted by Anam (2021), the APOS theory approach was used as an indicator of quadrilateral and triangle (TSS) assignment questions. In the TSS questions, there are 2 questions at the action stage, 1 question at the process stage, 1 question at the object stage, and 1 question at the scheme stage. With the results of this study, from 2 subjects of different genders, female students can carry out all four stages of APOS theory completely and optimally, so they are included in the category of having relational understanding. Meanwhile, male students have not fully carried out the stages in the APOS theory completely and optimally, especially at the action and object stages, so that the subjects are categorized as having an instrumental level of understanding.

Factors Influencing Students' Relational Understanding

Elementary and secondary school students' relational understanding is most influenced by students' problem-solving factors. One of them is in a study conducted by Linda et al. (2024) on 3 research subjects, it was found that the three subjects met three indicators of relational understanding. The indicators used in the study are indicators of relational understanding according to Baiduri which are adjusted to the stages of problem solving according to Polya. Table 4 presents the indicators of relational understanding in the problem solving used. The results of the study showed that the three subjects met all

stages of problem solving and indicators of relational understanding, so it can be said that the relational understanding possessed by students can help students in solving problems.

Table 4. Stages of Problem Solving and Indicators of Relational Understanding

Stages of Problem Solving	Indicators of Relational Understanding
Understand the problem	Students can construct relationships between components in a problem.
Make a plan	Students can construct relationships between previously learned mathematical structures or properties based on the problem
Implementing the plan	Able to solve problems based on relationships that have been built through previously learned mathematical
Checking back	structures or properties

(Linda et al., 2024)

This factor is also in line with empirical research conducted by (Sudrajat, 2022), which states that there is an influence between relational and instrumental understanding in mathematics learning in terms of solving mathematical problems.

4. CONCLUSION

From the analysis conducted on 11 articles, it can be concluded that research related to APOS theory and relational understanding limited to the period 2014 to 2024 with research subjects in elementary and secondary school students tends to have similarities in mathematics topics (numbers, algebra, and geometry). In articles discussing the APOS theory approach, the most frequently applied topic is numbers, even though the APOS theory approach itself was developed to be applied to abstract algebra, discrete mathematics, calculus, statistics, and other mathematics topics at the college level. Meanwhile, relational understanding is most widely applied to algebra topics. From the literature review that has been conducted, the APOS theory approach is most widely applied to question/test instruments. The APOS theory approach can be used as an indicator to identify categories of student understanding, including relational understanding. Meanwhile, the factor that most influences students' relational understanding of mathematics is the way students solve problems (problem solve).

From the results of this study, it is hoped that it can provide insight related to learning theory in mathematics, especially APOS theory, as well as its application in understanding and improving students' relational understanding abilities in mathematics learning. So that it can be a reference for educators in designing more effective learning strategies. In addition, this study also contributes to improving the quality of learning evaluation, where the APOS theory can be used as a basis for compiling evaluation instruments, especially in assessing students' conceptual understanding.

Findings regarding factors that influence relational understanding, namely students' ways of solving problems, also provide benefits for educators in determining more appropriate learning strategies.

6. RECOMMENDATION

Based on the findings and limitations of this study, future research is recommended to expand data sources beyond Scopus and Google Scholar in order to obtain a more comprehensive picture of studies related to APOS theory and relational understanding. Further studies should also involve broader educational levels, including senior high school and higher education, to examine the applicability of APOS theory across different levels of mathematical abstraction. Furthermore, future research is encouraged to investigate other factors that may impact students' relational understanding, such as learning environments, instructional strategies, and cognitive or metacognitive aspects, through empirical and longitudinal research designs.

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