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High School Students' Mathematical Problem-Solving Abilities: A Polya-Based Analysis

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Abstract
This study aims to analyze high school students' mathematical problem-solving abilities in exponential and logarithmic functions based on Polya's four stages of problem solving. This study employed a qualitative descriptive approach involving 72 tenth-grade students from a senior high school in Samarinda. Data were obtained through essay tests designed to reveal students' problem-solving processes and semi-structured interviews to strengthen the interpretation of students' work results. Data analysis was conducted through the stages of data reduction, data presentation, and conclusion drawing. The results showed that students' problem-solving abilities varied across ability categories. High-ability students tended to plan, execute, and review the entire problem-solving process, although they did not always write down the initial information explicitly. Students with moderate abilities showed an initial understanding of the problem but had difficulty devising and carrying out a plan, which led to inadequately verified final results. Meanwhile, students with low abilities experienced obstacles from the problem comprehension stage, which led to failure in the subsequent stages. These findings emphasize the importance of strengthening conceptual understanding and practicing systematic problem-solving steps in mathematics learning.

Keywords: mathematical problem-solving ability; Polya's four stages; exponential and logarithmic functions

Abstrak
Penelitian ini bertujuan untuk menganalisis kemampuan pemecahan masalah matematis siswa SMA pada materi fungsi eksponen dan logaritma berdasarkan empat langkah pemecahan masalah Polya. Penelitian ini menggunakan pendekatan deskriptif kualitatif dengan subjek 72 siswa kelas X dari salah satu SMA Negeri di Kota Samarinda. Data diperoleh melalui tes uraian yang dirancang untuk mengungkap proses pemecahan masalah siswa serta wawancara semi-terstruktur untuk memperkuat interpretasi terhadap hasil pekerjaan siswa. Analisis data dilakukan melalui tahapan reduksi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa kemampuan pemecahan masalah siswa bervariasi antar kategori kemampuan. Siswa berkemampuan tinggi cenderung mampu merencanakan, melaksanakan, dan memeriksa kembali penyelesaian masalah secara utuh, meskipun tidak selalu menuliskan informasi awal secara eksplisit. Siswa berkemampuan sedang menunjukkan pemahaman awal terhadap masalah, tetapi mengalami kesulitan pada tahap perencanaan dan pelaksanaan sehingga hasil akhir tidak tervalidasi dengan baik. Sementara itu, siswa berkemampuan rendah mengalami hambatan sejak tahap memahami masalah, yang berdampak pada kegagalan pada tahap-tahap berikutnya. Temuan ini menegaskan pentingnya penguatan pemahaman konseptual dan pembiasaan langkah pemecahan masalah secara sistematis dalam pembelajaran matematika.

Kata Kunci: kemampuan pemecahan masalah matematis; empat tahap Polya; fungsi eksponen dan logaritma

1. INTRODUCTION

Education plays a strategic role in human resource development and the attainment of twenty-first-century competencies. One of its primary aims is to equip students with higher-order thinking skills, including critical thinking, creative thinking, problem-solving, and mathematical reasoning (Ramadhanti et al., 2022). In this context, mathematics education makes a significant contribution to the development of these abilities. Learning mathematics requires precision, logical reasoning, and the ability to model and solve problems systematically. Therefore, mathematical problem-solving ability serves as one of the key indicators of successful mathematics learning (Liljedahl et al., 2016).

In learning practice, problem solving is not merely the application of procedures, but rather a series of cognitive processes: understanding the problem, designing strategies, implementing solutions, and reflecting on the results (Usman et al., 2023). This process aligns with the characteristics of deep learning, which emphasize conceptual understanding, connections among ideas, and the ability to transfer knowledge to new contexts (Liljedahl et al., 2016; Weng et al., 2023). These characteristics are consistent with the direction of learning promoted in the Merdeka Curriculum, which encourages students to understand and apply knowledge meaningfully. Through problem-solving activities, students actively construct mathematical meaning and connect previously learned concepts with various situations they encounter. Thus, learning extends beyond procedural mastery toward a deeper level of understanding (Jäder & Johansson, 2025). Therefore, positioning problem solving at the core of learning activities has the potential to support deep and sustainable learning.

At the high school level, exponential and logarithmic functions require a strong grasp of concepts and algebraic manipulation, and often serve as a foundation for more advanced topics. However, research shows that many students experience conceptual and procedural difficulties with these topics, such as understanding the relationship between exponents and logarithms and applying their properties correctly, which leads to errors in problem solving (Campo-Meneses et al., 2021; Kuper & Carlson, 2020). In previous studies in a similar learning context, it was found that students' errors in solving exponent and logarithm problems often stemmed from conceptual weaknesses, namely a tendency to rely on memorized rules without adequate conceptual understanding, so that an initial incorrect model led to seemingly correct algebraic steps that still produced incorrect answers (Ramadhanti, 2025). In addition, students often misinterpret the information in the questions, choose inappropriate strategies, and neglect to double-check their solutions (Putri et al., 2024; Zaenuri & Astutiningtyas, 2023). This condition emphasizes the need to focus on students' thinking processes in learning exponents and logarithms.

The results of initial observations and interviews in the classroom showed that the 10th-grade mathematics teacher, who was the subject of this study, reported that the average scores of students' daily tests on exponential and logarithmic functions had not yet reached the minimum passing criteria. In addition, students tended not to write down the information in the questions completely and did not recheck their answers. The learning practices observed show that teachers model problem solving sequentially, starting from guiding students to understand the problem, devise a plan, carry out the plan, and look back at the obtained solutions. This learning pattern is in line with the problem-solving stages proposed by Polya, thus providing the empirical basis for the selection of this framework in the study.

Theoretically, Polya's problem-solving framework has the strength to represent students' thinking processes in a sequential and reflective manner, because it not only emphasizes the final result, but also traces the cognitive stages that students go through during problem solving. These stages include: (1) understanding the problem, which involves identifying known and unknown information; (2) devising a plan, which involves determining relevant approaches or methods; (3) carrying out the plan, which involves systematically applying the chosen strategy; and (4) looking back, which involves evaluating the procedures and solutions obtained (Usta, 2020). With these characteristics, Polya's framework is relevant for comprehensively analyzing students' mathematical problem-solving abilities, particularly in the subject matter of exponential and logarithmic functions, which require conceptual understanding and procedural accuracy.

Numerous studies have shown that exponential and logarithmic functions are challenging topics for secondary school students because they require conceptual understanding, procedural accuracy, and the ability to connect different mathematical representations (Campo-Meneses et al., 2021; Kuper & Carlson, 2020). However, these studies have mainly focused on conceptual difficulties, mathematical connections, or the characteristics of students' understanding of exponential and logarithmic concepts. They have not specifically examined how students progress through each stage of problem solving when working on tasks involving these topics. Although Polya's framework has been widely used to analyze mathematical problem-solving abilities, its application has been more common in other mathematical contexts or in studies emphasizing instructional effectiveness rather than providing an in-depth description of students' problem-solving processes in exponential and logarithmic functions (Chacón-Castro et al., 2023). Therefore, further research is needed to specifically analyze tenth-grade students' mathematical problem-solving abilities in exponential and logarithmic functions based on Polya's four stages, namely understanding the problem, devising a plan, carrying out the plan, and looking back. This study addresses this gap by describing students' problem-solving processes across different ability categories, thereby identifying the stages at which students demonstrate strengths or encounter difficulties when solving exponential and logarithmic problems.

Based on this description, it is important to conduct a study that focuses on students' mathematical problem-solving abilities in exponential and logarithmic functions using Polya's four-stage framework. This study aims to describe how each stage of problem-solving is manifested in the problem-solving process by 10th-grade students at a public high school in Samarinda. By analyzing students' thinking processes at each stage of problem solving, this study is expected to provide an empirical picture of the patterns of abilities and obstacles experienced by students, as well as a basis for teachers to consider in designing mathematics learning that better supports the development of conceptual understanding and meaningful problem-solving skills, especially in the subject matter of exponential and logarithmic functions.

2. RESEARCH METHOD

The research method used in this study was qualitative descriptive research, which was chosen because it aims to describe and describe in depth students' mathematical problem-solving abilities as they naturally arise in the learning process, without treating or manipulating the variables under study (Hall & Liebenberg, 2024; Kim et al., 2017). The participants were 72 tenth-grade students from a senior high school in Samarinda. The focus of the study was their mathematical problem-solving abilities, analyzed through Polya's four stages: understanding the problem, devising a plan, carrying out the plan, and looking back. These stages served as the framework for analyzing the students' thinking processes. (Chacón-Castro et al., 2023).

The research instruments used included observation guidelines, a written test consisting of five essay questions, and interview guidelines. The observation guidelines were used to record how teachers modeled and guided students in problem solving. The essay test on exponential and logarithmic functions was designed to reveal students' conceptual understanding, procedural skills, and reasoning processes, and was completed without calculators so that students' thought processes could be traced. The interview guidelines were used to obtain supporting information about students' problem-solving abilities in a flexible manner while remaining relevant to the research context. The quality of the instruments was ensured through validation and testing in classes that were not subject to the research, including item analysis (difficulty index, discriminating power, and reliability). The results of the test analysis showed that all items had a moderate level of difficulty, were able to distinguish between students' abilities, and had high reliability, so that all items were declared feasible and used in the research.

Data collection techniques included written tests, classroom observations, and interviews. The tests measured students' problem-solving abilities, observations recorded the learning process and how teachers guided students, and interviews were conducted with representatives from each ability category (high, moderate, low) using pseudonyms to maintain anonymity. The analysis followed the stages of transcription, reduction, presentation, conclusion drawing, and verification; at the reduction stage, students' answers were analyzed according to Polya's four stages and relevant data were presented descriptively according to ability category. Students' responses were scored based on their achievement of Polya's four problem-solving stages, and the scores were converted to a scale of 0 to 100 to determine the ability categories. The resulting score was denoted by (x) and classified into high, moderate, and low categories according to the score ranges presented in Table 1. In addition, the percentage of achievement at each of Polya's stages was calculated to describe students' mathematical problem-solving abilities proportionally.

Tabel 1. Student ability categories based on test scores

Score Range	Category
$80 \le x \le 100$	High
$60 \le x < 80$	Moderate
$0 \le x < 60$	Low

3. RESULT AND DISCUSSION

3.1 Result

This study reveals and describes problem-solving abilities in exponential and logarithmic functions based on Polya's four stages of problem solving, namely understanding the problem, devising a plan, carrying out the plan, and looking back. The test was conducted in writing for 3×40 minutes with 5 essay questions. The test scores were used to categorize students' abilities into high, moderate, and low, as summarized in Table 2.

Table 2. Student Ability Category Results

Category	Frequency	Percentage
High	4	5,6%
Moderate	27	37,5%
Low	41	56,9%
Total	72	100%

Table 2 shows that most students were classified in the low-ability category, comprising 41 students or 56.9% of the participants. A total of 27 students, or 37.5%, were classified in the moderate-ability category, while only 4 students, or 5.6%, were classified in the high-ability category. This distribution indicates that students' mathematical problem-solving abilities were generally dominated by the low-ability category.

In addition, to determine students' overall problem-solving ability, the percentage of achievement for each of Polya's four problem-solving stages, namely understanding the problem (P1), devising a plan (P2), carrying out the plan (P3), and looking back (P4), was calculated and is presented in Table 3.

Table 3. Percentage of Problem-Solving Ability Based on Polya's Problem-Solving Stages

Question	Percentage of Polya's Problem-Solving Stages							
	P1	Category	P2	Category	P3	Category	P4	Category
1	60,19%	Moderate	63,19%	Moderate	52,31%	Low	56,94%	Low
2	60,65%	Moderate	59,72%	Low	51,85%	Low	52,78%	Low
3	45,83%	Low	43,75%	Low	37,96%	Low	28,47%	Low
4	60,65%	Moderate	49,31%	Low	45,83%	Low	44,44%	Low
5	72,22%	Moderate	77,08%	Moderate	78,70%	Moderate	64,58%	Moderate
Average	59,91%	Low	58,61%	Low	53,33%	Low	49,44%	Low

Based on Table 3, the average achievement across all of Polya's problem-solving stages fell within the low category. Understanding the problem recorded the highest percentage at 59.91%, followed by devising a plan at 58.61%, carrying out the plan at 53.33%, and looking back at 49.44%. This pattern indicates a gradual decline in achievement from the initial stage to the final stage of problem solving. Among the five questions, Question 5 showed relatively better performance because students' achievement across all four stages was classified in the moderate category.

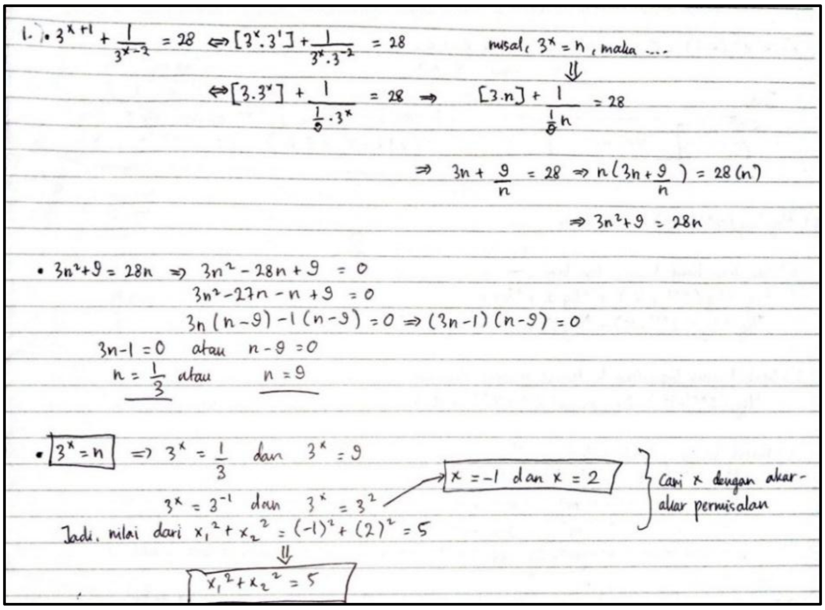
An in-depth analysis is presented using excerpts from students' responses to different questions, which were selected to represent the problem-solving characteristics of students in each ability category. The questions were not selected to compare their levels of difficulty, but to highlight students' thinking patterns across Polya's stages as reflected in different questions. All questions used in this study underwent difficulty analysis and were classified as having a moderate level of difficulty. Therefore, the differences in problem-solving characteristics identified in the following analysis were considered to reflect differences in students' abilities rather than differences in question difficulty. The results of the mathematical problem-solving ability test on exponential and logarithmic functions, analyzed based on Polya's four problem-solving stages, are presented according to three student ability categories: high, moderate, and low.

3.1.1 Description of Test Results for Students in the High Ability Category

In the high-ability category, S1's response to Question 1 was selected because its written solution pattern was consistent with those of the other students in this category. S1's written work is presented in Figure 1.

Diketahui x_1 dan x_2 merupakan akar-akar dari persamaan $3^{x+1} + \frac{1}{3^{x-2}} = 28$. Tentukan nilai dari $x_1{}^2 + x_2{}^2$.

Translation:
Given that x_1 and x_2 are the roots of the equation $3^{x+1} + \frac{1}{3^{x-2}} = 28$. Determine the value of $x_1{}^2 + x_2{}^2$.



1. • $3^{x+1} + \frac{1}{3^{x-2}} = 28 \Leftrightarrow [3^x.3^1] + \frac{1}{3^x.3^{-2}} = 28$ misal, $3^x = n$, maka ....

$\Leftrightarrow [3.3^x] + \frac{1}{\frac{1}{9}.3^x} = 28 \Rightarrow [3.n] + \frac{1}{\frac{1}{9}n} = 28$

$\Rightarrow 3n + \frac{9}{n} = 28 \Rightarrow n(3n + \frac{9}{n}) = 28(n)$

$\Rightarrow 3n^2 + 9 = 28n$

• $3n^2 + 9 = 28n \Rightarrow 3n^2 - 28n + 9 = 0$

$3n^2 - 27n - n + 9 = 0$

$3n(n-9) - 1(n-9) = 0 \Rightarrow (3n-1)(n-9) = 0$

$3n - 1 = 0$ atau $n - 9 = 0$

$n = \frac{1}{3}$ atau $n = 9$

• $\boxed{3^x = n} \Rightarrow 3^x = \frac{1}{3}$ dan $3^x = 9$

$3^x = 3^{-1}$ dan $3^x = 3^2$ $\rightarrow$ $\boxed{x = -1 \text{ dan } x = 2}$ Cari x dengan akar-akar permisalan

Jadi, nilai dari $x_1{}^2 + x_2{}^2 = (-1)^2 + (2)^2 = 5$

$\Downarrow$

$\boxed{x_1{}^2 + x_2{}^2 = 5}$

Figure 1. Representative work of a high-ability student (S1).

At the first stage, understanding the problem, Figure 1 shows that S1 did not explicitly write down the given and required information. However, during the interview, S1 was able to verbally identify the relevant information, indicating that the student understood the problem well. The following interview excerpt supports this interpretation.

P: "Okay, after reading the question, do you still remember the steps to solve it?"
S1: "I still remember, Miss."
P: "Do you understand what the question means? If so, explain it to me so that I can be sure you understand."
S1: "I understand, Miss. The question gives us this exponential equation, Miss, asking for x_1 *and* x_2*, and then asking us to determine the value of* $x_1{}^2 + x_2{}^2$*."*
P: "Okay, but why didn't you write down the information given and asked in the question on the answer sheet?"
S1: "Yes, Miss, I didn't write it down, but I understand, Miss."

Based on the interview excerpt, S1 demonstrated an understanding of the problem and the required solution, although this information was not written on the answer sheet. Furthermore, at the devising a plan stage, the written work presented in Figure 1 shows that S1 was able to identify the relevant exponent property needed to solve the problem. This indicates that S1 was able to devise and document an appropriate solution plan.

Next, at the carrying out the plan stage, Figure 1 shows that S1 was able to determine the roots of the exponential equation and present the solution steps sequentially, leading to the correct final answer. This indicates that S1 was able to carry out the plan successfully. At the final stage, looking back, S1 wrote an appropriate conclusion and substituted the obtained values back into the equation to verify the correctness of the solution. The following interview excerpt confirms this action.

P: "After completing the calculations, what did you do?"
S1: "I wrote down the conclusion, then I double-checked it, Miss. I checked the values of x_1 and x_2 that I had obtained, substituted them into the exponential equation to see if they were correct, and found that they were, Miss. I was finally relieved, Miss."

Overall, S1 demonstrated mastery of Polya's four problem-solving stages. Although the given and required information was not written on the answer sheet, S1 understood the problem, devised an appropriate plan, carried out the plan systematically, and looked back at the solution through substitution. Thus, S1's problem-solving process was comprehensive and verified.

3.1.2 Description of Test Results for Students in the Moderate Ability Category

S2's response to Question 4 was selected as representative of the responses of students in the moderate-ability category. S2's written work is presented in Figure 2.

Tentukan himpunan penyelesaian dari ${}^{\frac{1}{5}}log(x+\sqrt{3}) + {}^{\frac{1}{5}}log(x-\sqrt{3}) < 0$.

Translation:
Determine the solution set of ${}^{\frac{1}{5}}log(x+\sqrt{3}) + {}^{\frac{1}{5}}log(x-\sqrt{3}) < 0$.

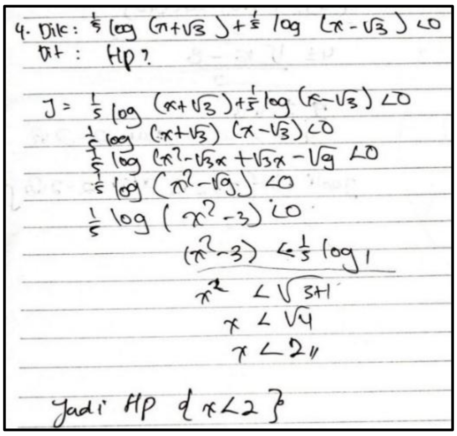

4. Dik: ${}^{\frac{1}{5}}\log(x+\sqrt{3}) + {}^{\frac{1}{5}}\log(x-\sqrt{3}) < 0$
Dit: Hp?
J = ${}^{\frac{1}{5}}\log(x+\sqrt{3}) + {}^{\frac{1}{5}}\log(x-\sqrt{3}) < 0$
${}^{\frac{1}{5}}\log(x+\sqrt{3})(x-\sqrt{3}) < 0$
${}^{\frac{1}{5}}\log(x^2-\sqrt{3}x+\sqrt{3}x-\sqrt{9}) < 0$
${}^{\frac{1}{5}}\log(x^2-\sqrt{9}) < 0$
${}^{\frac{1}{5}}\log(x^2-3) < 0$
$(x^2-3) < {}^{\frac{1}{5}}\log 1$
$x^2 < \sqrt{3+1}$
$x < \sqrt{4}$
$x < 2$
Jadi Hp $\{x < 2\}$

Figure 2. Representative work of a moderate-ability student (S2).

At the understanding the problem stage, it can be seen from the written test results in Figure 2 that S2 wrote down the information given and asked in the problem. Thus, S2 formally demonstrated an initial understanding of the problem. The following interview excerpt confirms that S2 understood the problem.

P: "Do you understand what the question means?"
S2: "Yes, Miss."
P: "Try to explain it so that I can believe that you understand."
S2: "This is a logarithmic inequality, Miss, and we are asked to find the solution set."

During the devising a plan stage, S2 did not clearly write the solution plan on the answer sheet. S2 also did not explicitly formulate the domain conditions, particularly the numerus requirements, when applying the properties of logarithms. This indicates that

S2 was unable to devise an appropriate solution plan. This finding was confirmed by the interview results.

P: "After understanding the question, what do you do next?"

S2: "I wrote down the answer, ${}^{\frac{1}{5}}log(x+\sqrt{3}) + {}^{\frac{1}{5}}log(x-\sqrt{3})$. *I remembered that the property of logarithms is that when you add, you multiply, so the form becomes* ${}^{\frac{1}{5}}log(x+\sqrt{3}).(x-\sqrt{3})$.

P: "Do you remember the conditions that need to be written down when solving logarithmic inequalities?"
S2: "Hmm, I didn't think of that at the time, Miss."
P: "What is the condition for the numerus?"
S2: "Greater than zero, right, Miss?"
P: "Yes, that's right. Then why wasn't it written on the answer sheet?"
S2: "I was confused when I was working on it yesterday, Miss."

The interview excerpt confirms that S2's plan was not comprehensive, particularly because the domain requirements crucial to logarithmic problems were not considered. The next stage was carrying out the plan. As shown in Figure 2, S2 was able to apply the properties of logarithms correctly but became stuck in the subsequent steps, which led to calculation errors and prevented the student from obtaining the correct final answer. To clarify the carrying out the plan stage, the following interview excerpt is presented.

P: "Okay, you're correct up to this step. What's the next step?"
S2: "To be honest, Miss, I don't understand the next step, so I worked with my friend a little."
P: "Do you understand when writing ${}^{\frac{1}{5}}log\ 1$ *?"*
S2: "No, Miss, I forgot."
P: "Based on the $\frac{1}{5}$ *problem, do you still remember how it should be?"*
S2: "Ohhh yes, the sign is reversed, Miss, it should be greater than 0."

The final stage was looking back. S2 wrote a conclusion, but the result was inaccurate. Based on the interview, S2 reported attempting to look back at the solution but remained confused. The following interview excerpt illustrates S2's reflections during the looking back stage.

P: "What did you do next?"
S2: "I wrote down the solution set, then continued working on the last question, Miss."
P: "Did you double-check your calculations?"
S2: "Actually, I looked at it again because I wasn't sure about my answer, but I was still confused, Miss."

Based on S2's work in the moderate-ability category, a clear pattern emerged. S2 was able to understand the problem well but was unable to devise an appropriate plan, carry out the plan correctly, or look back at the solution effectively.

3.1.2 Description of Test Results for Students in the Low Ability Category

The low-ability category was represented by S3's work on Question 5. S3's written work is presented in Figure 3.

Jumlah koloni bakteri $N(t)$ merupakan model fungsi eksponensial $N(t) = 1000 \times 2^{kt}$, dengan $0 < k < 1$ dan $t \geq 0$, dimana t diukur dalam hari. Setelah 5 hari jumlah koloni bakteri adalah 2000. Tentukan jumlah koloni bakteri setelah 15 hari.

Translation:

The number of bacterial colonies $N(t)$ is modeled by the exponential function $N(t) = 1000 \times 2^{kt}$, where $0 < k < 1$ and $t \geq 0$, and t is measured in days. After 5 days, the number of bacterial colonies is 2000. Determine the number of bacterial colonies after 15 days.

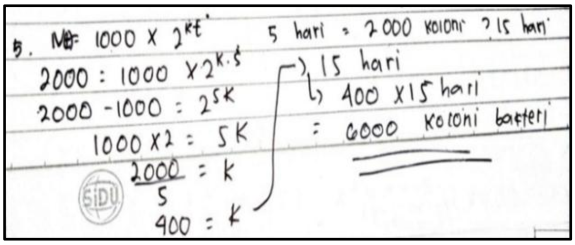

5. N(t) = 1000 x 2^kt   5 hari = 2000 koloni ? 15 hari
2000 : 1000 x 2^k.5   15 hari
2000 - 1000 : 2^5K   400 x 15 hari
1000 x 2 : 5K   : 6000 koloni bakteri
2000/5 : k
400 : k

Figure 3. Representative work of a low-ability student (S3).

At the first stage, understanding the problem, Figure 3 shows that S3 did not write down the given and required information on the answer sheet. During the interview, S3 reported being confused by the information presented in the problem and was only able to identify some of it hesitantly. Therefore, S3 was considered unable to understand the problem adequately. This interpretation is supported by the following interview excerpt.

P: "Okay, after reading the question, do you still remember the steps to solve it?"
S3: "Pretty well, Miss..."
P: "Do you think this question is easy, medium, or difficult? Do you understand the question?"
S3: "It's difficult, Miss. Hmm, I'm still a little confused. I don't really understand it yet."
P: "What information is given in the question?"
S3: "It gives an exponential function, Miss. It is known that after 5 days, there are 2000 colonies, then we are asked to find the number of colonies after 15 days."
P: "Okay, you didn't write down the information that is known and asked in the question on the answer sheet? Why?"
S3: "Yes, Miss, I forgot, Miss."

Then, at the devising a plan stage, S3 did not write a detailed solution plan on the answer sheet. To clarify this finding, the following interview excerpt from S3 is presented.

P: "After understanding the question, what do you do next?"
S3: "I look for the value of k first, Miss."

S3 appeared to state only a general intention to determine the value of (k) without elaborating on the strategy or conditions underlying the exponential function model. Consequently, the solution plan written on the answer sheet was inadequate. As a result, S3 encountered difficulties at the carrying out the plan stage. S3 made errors in algebraic manipulation and was unable to present the solution steps coherently, which prevented the student from obtaining the correct final answer. These findings are supported by the following interview excerpt.

P: "Okay, please explain to me how you solved it."
S3: "Well, from the information given in the question, t = 5 days when the number of colonies is 2000, so I put it into the exponential function to become $2000 = 1000 \times 2^{k5}$. *Then, because it moved to the other side of the equation, it became* $2000 - 1000 = 2^{5k}$.
P: "Then what?"
S3: "After getting the value of k, which is 400, I think that's the number of colonies per day. Because the question asks for the number of colonies after 15 days, I multiplied it by 400×15."
P: "Are you sure about your answer?"
S3: "No, Miss, I think it's wrong, because on the right side of the multiplication, you can't move the terms... is this divided, Miss? Eh, I don't know, Miss."
P: "If (while writing) $6 = 2 \times k$. *Can you determine the value of k?"*
S3: "3, Miss, because 6 divided by 2 is equal to 3. Oh, yes, that means it should be $\frac{2000}{1000} = 2^{5k}$.
P: "Then how do you find the value of k if you know that $\frac{2000}{1000} = 2^{5k}$*?*
S3: "It becomes $2 = 2^{5k}$*, Miss. Then how, Miss, I don't know."*

The final stage is looking back. S3 was unable to draw an appropriate conclusion, and the final answer did not correspond to the context of the problem. The following interview excerpt confirms that S3 did not look back because the student did not understand the solution process.

P: "After getting the final result, what did you do next?"
S3: "Nothing, Miss, that's it."
P: "Did you double-check the calculations?"
S3: "No, Miss, there's no point in checking because I don't really understand it."

Thus, S3 experienced substantial difficulties across all four stages of Polya's problem-solving process. The difficulty began at the understanding the problem stage, continued through devising a plan and carrying out the plan, and culminated in the absence of looking back or verifying the solution.

3.2 Discussion

The findings indicate that students' achievement across Polya's problem-solving stages tended to decline from understanding the problem to looking back, with all stages remaining in the low category. At the first stage, understanding the problem, most students did not fully write down the given and required information. However, the interview results showed that high- and moderate-ability students were generally able to explain the meaning of the problem, whereas low-ability students still showed uncertainty in identifying the information presented. At the next stage, devising a plan, some students proceeded directly to calculations without stating the strategy they intended to use. High-ability students were generally able to formulate an appropriate plan, moderate-ability students developed incomplete plans, and low-ability students tended to rely on trial-and-error approaches.

At the carrying out the plan stage, high-ability students were generally able to apply their chosen strategies systematically and in accordance with the relevant concepts. Moderate-ability students attempted to follow the planned solution steps but still made errors in calculations and in applying the properties of exponential and logarithmic functions. Meanwhile, low-ability students encountered difficulties from the beginning of the solution process, resulting in unsystematic steps that often did not lead to correct answers. At the final stage, looking back, differences among the ability categories became more apparent. High-ability students generally checked their results, reviewed their solution steps, and wrote conclusions that were consistent with the questions posed. Moderate-ability students began to show efforts to verify their results, but their checking was not yet comprehensive, leading to conclusions that were inaccurate or inconsistent with their calculations. In contrast, low-ability students generally did not look back because they lacked confidence in their solution processes or had not obtained correct answers. These differences indicate variations in students' problem-solving processes across the three ability categories.

These variations in the problem-solving process indicate that success at each of Polya's stages is interconnected and plays an important role in students' ability to solve mathematical problems. Success at the initial stage, understanding the problem, strongly influences students' progress through the subsequent stages, namely devising a plan, carrying out the plan, and looking back. High-ability students clearly demonstrated this pattern. Although they did not write down the given and required information on the answer sheet, their understanding of the problem was evident. Consequently, they were able to devise appropriate plans, carry out the plans accurately, and look back at their solutions systematically, leading to correct and verified final answers. This finding is consistent with previous studies showing that students with a strong conceptual understanding of the problem structure are able to devise appropriate plans and perform meaningful verification (Riyadi et al., 2021).

In contrast, students in the moderate-ability category displayed a different pattern. They were able to complete the understanding the problem stage, but their performance at the devising a plan stage was inadequate, particularly because they ignored the domain conditions of logarithmic functions. As a result, although some subsequent procedures were performed correctly, the solution process eventually stopped, and the students were uncertain about looking back at their answers. This finding is consistent with previous studies showing that difficulties in devising a plan, such as ignoring domain conditions and numerus requirements, are often a major source of errors in exponential and logarithmic problems, even when the carrying out the plan stage appears correct in several steps (Campo-Meneses et al., 2021; Kuper & Carlson, 2020). This indicates that students' errors are rooted in conceptual weaknesses rather than merely procedural difficulties. Consequently, procedures that appear correct may still produce incorrect answers when the initial plan is inappropriate (Ramadhanti, 2025).

In the low-ability category, the difficulties were cumulative. Students' inability to understand the information presented in the problems hindered their ability to devise appropriate plans, which subsequently led to errors in carrying out the plans and prevented them from reaching the looking back stage. This finding is consistent with previous research indicating that difficulties in understanding the problem may become an initial obstacle that affects the subsequent stages of problem solving (Wahab A et al., 2024). The interconnection among these stages also supports the view that instructional interventions that systematically incorporate Polya's four stages should not be treated merely as routine procedures, but as a thinking framework that facilitates reflection, strengthens connections among the stages, and reduces conceptual and procedural errors (Chacón-Castro et al., 2023)

The findings of this study indicate that strengthening conceptual understanding should be a primary focus because difficulties at the early stages of problem solving tend to affect the subsequent stages. Furthermore, familiarizing students with identifying the given and required information, considering the conditions that need to be checked, and systematically looking back at their solutions may help them progress more smoothly across Polya's problem-solving stages and, consequently, improve their mathematical problem-solving abilities.

4. CONCLUSION

The findings of this study indicate that students' mathematical problem-solving abilities in exponential and logarithmic functions are closely related to the integration of Polya's problem-solving stages. High-ability students demonstrated consistent connections among understanding the problem, devising a plan, carrying out the plan, and looking back, resulting in a more focused and systematic problem-solving process. In contrast, moderate- and low-ability students experienced difficulties at the early stages, particularly in conceptual understanding and devising a plan. These difficulties hindered their progress through the subsequent stages and led to inaccurate solutions. The findings confirm that successful mathematical problem solving depends not only on the ability to perform calculations but also on conceptual understanding, the ability to devise appropriate solution plans, and awareness of the thinking process involved at each stage. From a practical perspective, these findings can serve as a basis for mathematics teachers to design instruction that familiarizes students with identifying relevant information in a problem, devising a plan before performing calculations, carrying out the plan systematically, and looking back at their solutions, particularly to support moderate- and low-ability students.

5. RECOMMENDATION

Based on the findings of this study, future research may develop and examine instructional strategies that systematically strengthen each of Polya's problem-solving stages, particularly in the context of exponential and logarithmic functions. Classroom strategies may include guiding questions to help students identify the given and required information, structured worksheets to support the formulation of a plan before performing calculations, and reflective activities at the end of the solution process to evaluate the accuracy of procedures and the appropriateness of answers in relation to the problem context. Gradual instructional support may also be considered to help moderate- and low-ability students develop more independent problem-solving strategies. Similar studies may be extended to other mathematical topics with complex conceptual and procedural demands to examine the consistency of students' problem-solving abilities across Polya's stages. Future research should also consider students' mastery of prerequisite concepts and their habits of documenting solution processes, as these factors may influence the findings.

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