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Mathematical connection profile of low-achieving students in pythagorean theorem problems

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Abstract

Mathematical connection ability is a fundamental competency that Indonesian students have not yet developed optimally, as evidenced by persistently low performance on both national and international assessments. This condition warrants in-depth qualitative investigation, particularly among low-achieving students whose specific connection difficulties remain poorly understood. This qualitative descriptive study was conducted at SMA Negeri 3 Semarang. A total of 72 tenth-grade students completed a preliminary written test measuring three NCTM mathematical connection indicators. Based on the results, ten students fulfilling at most one indicator were selected through purposive sampling as interview subjects. Data from semi-structured interviews were analyzed using the Miles and Huberman model. Results revealed nine subjects fulfilled only Indicator 1, one fulfilled only Indicator 2, and none fulfilled Indicator 3. Interview exploration revealed distinct thinking processes across all three indicators and three qualitative patterns of connection difficulty comprising weak prerequisite mastery, inability to link concepts that were individually understood, and inability to complete problem-solving despite having a sense of the solution path. These findings highlight the importance of diagnostic and instructional approaches tailored to the specific nature of each student's difficulty.

Keywords: mathematical connection; low-achieving students; NCTM indicators; Pythagorean theorem; qualitative research

Abstrak

Kemampuan koneksi matematis merupakan kompetensi fundamental yang belum dimiliki secara optimal oleh murid di Indonesia, sebagaimana tercermin dari rendahnya capaian pada asesmen nasional maupun internasional. Kondisi ini perlu dikaji secara mendalam, khususnya pada murid berkemampuan rendah yang karakteristik kesulitan koneksinya masih kurang dipahami. Penelitian deskriptif kualitatif ini dilaksanakan di SMA Negeri 3 Semarang. Sebanyak 72 murid kelas X mengikuti tes tertulis studi pendahuluan yang mengukur tiga indikator koneksi matematis NCTM. Berdasarkan hasilnya, sepuluh murid yang memenuhi paling banyak satu indikator dipilih melalui *purposive sampling* sebagai subjek wawancara. Data wawancara semi-terstruktur dianalisis menggunakan model Miles dan Huberman. Hasil menunjukkan sembilan subjek hanya memenuhi Indikator 1, satu subjek memenuhi Indikator 2, dan tidak ada yang memenuhi Indikator 3. Eksplorasi wawancara mengungkap proses berpikir yang berbeda pada ketiga indikator serta tiga pola hambatan koneksi: lemahnya penguasaan prasyarat, ketidakmampuan menghubungkan konsep-konsep yang sebenarnya sudah dipahami, dan ketidakmampuan menyelesaikan soal meskipun alur penyelesaiannya sudah tergambar. Temuan ini menegaskan pentingnya pendekatan diagnostik dan instruksional yang disesuaikan dengan karakteristik hambatan masing-masing murid.

Kata Kunci: koneksi matematis; murid berkemampuan rendah; indikator NCTM; Teorema Pythagoras; penelitian kualitatif

1. INTRODUCTION

Mathematical connection is one of the five process standards established by the National Council of Teachers of Mathematics (NCTM), alongside problem solving, reasoning and proof, communication, and representation (NCTM, 2000). Mathematical connections enable students to understand mathematics as an integrated body of knowledge rather than a collection of isolated procedures and formulas (Hiebert & Carpenter, 1992; NCTM, 2000; Hayagan, 2025). When students recognize and utilize connections among mathematical ideas, they develop deeper and more enduring understanding that supports further learning (Eli et al., 2011; Bingölbali & Coskun, 2016). Despite its recognized importance, Indonesian students continue to demonstrate persistently low mathematical connection abilities. The PISA 2022 results revealed that Indonesian students scored an average of 366 points in mathematics, significantly below the OECD average of 472 points, with only 18% attaining Level 2 proficiency (OECD, 2023).

Research on mathematical connections in Indonesia has grown substantially in recent years. Komalasari & Imami (2022) found that junior high school students' connection abilities on Pythagorean material were classified as moderate, with frequent conceptual and computational errors. Studies by Kenedi et al. (2019) and Hidayah et al. (2024) further confirmed that students across grade levels struggle to recognize inter-concept relationships and apply mathematical ideas in diverse contexts. García-García & Dolores-Flores (2021) emphasized that connections are not merely cognitive products but dynamic processes through which students construct mathematical meaning. However, the majority of existing studies have measured connection ability quantitatively or examined the effect of specific instructional models (Jahring, 2020; Suparti & Netriwati, 2021), with limited qualitative exploration of how students actually construct or fail to construct connections in multi-concept problem contexts.

A critical gap remains in the qualitative characterization of connection difficulties among low-achieving students. Most prior studies treat low performance as a uniform outcome, without distinguishing the qualitatively different barriers that lead to it. This study addresses that gap by investigating not merely whether students can connect concepts, but how their connection attempts break down and at what level. Specifically, it examines the Pythagorean theorem as a focal concept that necessarily integrates linear equations and distance-speed-time relationships which constitutes a multi-concept integration task that has received limited qualitative attention. Additionally, this study employs task-based interviews alongside written tests to surface connection potential that written assessment alone may miss, particularly for students affected by test anxiety or problem-format novelty.

In this study, mathematical connection ability is assessed based on three indicators adapted from NCTM (2000): (1) recognizing and using connections within a mathematical concept; (2) understanding how mathematical concepts are interrelated and build upon one another to form a coherent whole; and (3) recognizing and applying mathematical concepts in contexts outside mathematics. The research question guiding this study is: How is the mathematical connection profile of low-achieving high school students in solving Pythagorean theorem problems based on NCTM indicators?

2. RESEARCH METHOD

This study employed a qualitative descriptive research design. The research was conducted at SMA Negeri 3 Semarang during the 2025/2026 academic year, involving tenth-grade students. This school was selected because its student population includes a range of mathematical ability levels, allowing for the identification of low-achieving students with varied experiential backgrounds.

Participants were identified through a two-stage process. First, all 72 tenth-grade students across two classes completed a preliminary mathematical connection written test. Based on the results, ten students were selected through purposive sampling with the criterion of fulfilling at most one of the three NCTM mathematical connection indicators. Each subject is assigned a pseudonymous code (S1 through S10) to protect participant confidentiality. It is important to note that the determination of which indicator(s) each subject fulfilled is based entirely on written test performance and not on interview responses. The interview phase served solely as a qualitative exploration tool to describe the characteristics of students' connection difficulties in greater depth. The subject profiles are presented in Table 1.

Table 1. Subject Profiles Based on Preliminary Written Test

Code	Gender	Indicator Met	Score	Indicator 1	Indicator 2	Indicator 3
			/6	Connection within concept	Inter-concept connection	Real-world connection
S1	F	Indicator 1	2	✓	✗	✗
S2	F	Indicator 1	3	✓	✗	✗
S3	F	Indicator 2	3	✗	✓	✗
S4	M	Indicator 1	3	✓	✗	✗
S5	M	Indicator 1	3	✓	✗	✗
S6	M	Indicator 1	4	✓	✗	✗
S7	F	Indicator 1	3	✓	✗	✗
S8	F	Indicator 1	5	✓	✗	✗
S9	F	Indicator 1	4	✓	✗	✗
S10	M	Indicator 1	3	✓	✗	✗

Information:

✓ = fulfilled

✗ = not fulfilled

Two main instruments were used. The first was a mathematical connection test consisting of an integrated contextual problem involving a drone survey mission, requiring students to apply the Pythagorean theorem, linear equations, and distance-speed-time relationships within a single context. The second was a semi-structured interview guide organized into five sections: (1) mathematical learning experience, (2) thinking processes when solving the problem, (3) barriers and connection difficulties, (4) prerequisite knowledge, and (5) reflection and expectations. Both instruments were validated by expert validators prior to data collection.

Data collection was carried out in two phases: a written test followed by individual task-based interviews in which students explained their thinking processes while referring to their written work (Goldin, 2000). Data analysis followed the Miles and Huberman model: data condensation, data display, and drawing and verifying conclusions (Miles et al., 2020). Trustworthiness was ensured through method triangulation (comparing written tests with interview responses) and member checking.

3. RESULTS AND DISCUSSION

3.1 Results

3.1.1 Written Test Results

The preliminary written test results establish the foundational data of this study. Nine of ten subjects (S1, S2, S4, S5, S6, S7, S8, S9, S10) fulfilled only Indicator 1, with scores ranging from 2 to 5 out of 6. S3 represents a unique case as the only subject who fulfilled Indicator 2 rather than Indicator 1, with a score of 3/6. No subject fulfilled Indicator 3. These results confirm that all ten subjects meet the low-achieving criterion. Among subjects who fulfilled Indicator 1, notable score variation was observed: S8 achieved the highest score (5/6) while S1 achieved the lowest (2/6). This variation suggested that even within the same fulfilled indicator, meaningful qualitative differences in connection depth existed, which subsequent interviews were designed to explore.

3.1.2 Indicator 1: Connections Within a Single Concept

Nine subjects fulfilled Indicator 1 on the written test, yet their interview responses revealed substantial differences in the quality of that fulfillment. Three patterns of thinking emerged across these subjects. The first pattern, found in S1, S4, S5, and S6, was concept recognition at a purely lexical or visual level. These subjects could identify surface features of the problem but could not articulate the mathematical concepts involved or describe how to apply them. When asked what concepts came to mind upon reading the problem, S1 could only identify isolated words.

R : "When you read the problem, what mathematical concepts or topics did you remember?"
S1 : "Average speed and distance, Sir. I forgot the exact term, but it was about speed and distance."

R : "What were the steps you used to solve this problem?"
S1 : "...I did not quite understand it, so I just guessed."

S1 recognized only the words "speed" and "distance" in the problem but could not name the relevant mathematical concept or describe any solution strategy. S4 showed a similar pattern, he could identify Pythagorean theorem from the directional configuration but was unable to progress beyond that initial recognition. Meanwhile, S5 demonstrated a more specific barrier at the variable level, expressing genuine confusion about why mathematics uses letters at all.

R : "From this problem, which part was the most confusing?"
S5 : "That X part, Sir. What is that letter, X? Why does mathematics have to use letters? Why not just use numbers?"

S6, despite fulfilling Indicator 1 on the test with a score of 4/6, was unable to identify any concept during the interview, explaining that years of absence from school due to athletic dispensations had left his middle school knowledge severely fragmented.

R : "What mathematical concepts or topics did you remember when you saw this problem?"
S6 : "Nothing came to mind, honestly, none. There were many concepts from middle school that I did not learn because I was often absent."

The second pattern, observed in S2, S7, and S10, was partial concept recognition anchored by visual cues. These subjects spontaneously identified the Pythagorean theorem based on the directional configuration of the problem, and partially recalled distance-speed-time relationships as memorized facts. S2 recognized the Pythagorean theorem through the visual cue of perpendicular directions.

R : "When you first saw this problem, what concept did you think was used?"
S2 : "Because there are directions, it means using... some kind of triangle. I mean the Pythagorean theorem, Sir."

S10 revealed a memorization-based strategy that could not accommodate novel formats.

R : "In your opinion, what is the cause of that difficulty?"
S10: "Because I rely more on memorization, Sir. So I know the formulas, but I do not understand why those formulas are used. If the problem changes slightly, I do not know how to adapt."

S7 similarly identified the Pythagorean theorem through the visual cue of perpendicular directions but did not spontaneously recall linear equations or distance-speed-time as relevant concepts. She described mathematics as fundamentally confusing due to its variability across problem types.

The third pattern, observed in S3, S8, and S9, was multi-concept recognition with richer conceptual engagement. S3 approached the problem by first analyzing directional geometry and then identifying the need for equations and substitution. S8 identified distance-speed-time, coordinates, and compass directions before identifying the Pythagorean theorem, suggesting a more systematic reading of the problem. Meanwhile S9 demonstrated the richest spontaneous recognition, identifying Pythagorean theorem from directional cues and connecting it to distance-speed-time. However, her recognition was immediately accompanied by panic.

R : "What mathematical concepts or topics did you remember when seeing this problem?"
S9 : "Pythagoras, Sir. Because the directions east and north immediately made me picture a right triangle. Then there were speed and time, so I also remembered the distance formula."
R : "Are there any other related concepts?"
S9 : "It seems there are also equations because the speed is still in X. But at that time, I did not think about that, I panicked right away."

Among all nine subjects who fulfilled Indicator 1, these three showed the deepest qualitative engagement, though even they faced barriers when attempting to integrate the concepts.

3.1.3 Indicator 2: Inter-Concept Connections

No subject other than S3 fulfilled Indicator 2 on the written test. Interview exploration, however, revealed that subjects' difficulties with inter-concept connection were not uniform. Three qualitatively distinct thinking patterns emerged across the ten subjects. Subjects S1, S4, S5, and S6 showed no meaningful attempt at inter-concept connection. Their thinking did not progress beyond individual concept recognition, and in most cases did not reach even that. S1 described her difficulty in connecting concepts as a fundamental inability to find the thread between steps. This difficulty was also evident in S4's interview response.

R : "Did you experience difficulties in connecting those concepts?"
S4 : "Yes, Sir. I do not necessarily understand even a single concept, let alone when they have to be combined."

For S5, the barrier was even more fundamental that is the inability to work with variables meant that any step requiring algebraic representation was blocked from the outset. S6 confirmed that across all of middle school, the prerequisite concepts were never adequately learned.

R : "If it is concept by concept, like linear equations, speed-distance-time, Pythagoras, do you understand them?"
S6 : "No, Sir. I completely missed all the middle school material. In 7th grade, I had constant dispensations, in 8th grade I was still taking it easy, and only in 9th grade was I pushed."

Subjects S2, S7, and S10 showed a different pattern. They understood individual concepts but could not determine how to sequence them for integrated problem-solving. S2 recognized that the problem required multiple interconnected steps, yet she was unable to determine which procedure should be carried out first. She also noted that the need to find intermediate values before reaching the final answer was confusing. Meanwhile, S10 identified the specific missing link most precisely.

R : "Did you experience difficulties in connecting those concepts?"
S10: "Yes, Sir. I know each of the formulas. But I do not know the order to use them. Where to start, which one to use first, that is what I cannot do."
R : "Which part was the most difficult to connect?"
S10: "From the equations to distance-speed-time, Sir. I did not know that the speed had to be found first using the equation before I could calculate the distance."

S7 described knowing the concepts separately but being unable to trace the path from one to another. When the researcher illustrated with an example involving Pythagorean theorem and area.

R : "For example, if there is a problem asking you to calculate the area, where you have to use Pythagoras first, and then calculate the area. Could you do it?"
S7 : "I know the area formulas, for example, length times width or base times height divided by two. But how to get from Pythagoras to the area, that is what I do not understand."

Subjects S3, S8, and S9 showed awareness of how the concepts relate but faced specific execution barriers. S8 articulated her understanding of individual concepts yet acknowledged feeling lost when they needed to be combined.

R : "Is the confusion because you do not yet know how to determine the steps, or do you not understand the formulas?"
S8 : "I understand the basic formulas. How to link them is what I am still confused about. This is the first time I have encountered such a highly complex problem. Usually, problems only connect one or two concepts, but this one has many."

S9 expressed a similar awareness alongside an affective barrier. She possessed adequate conceptual knowledge but was paralyzed by the fear of choosing the wrong sequence.

R : "Did you experience difficulties in connecting those concepts?"
S9 : "Yes, Sir. If it is each concept individually, I can do it, but when asked to combine them, I am not sure about the sequence of steps. I am afraid that if the sequence is wrong, the entire answer will be wrong."

Meanhile S3, who was the only subject to fulfill Indicator 2 on the test, described a structured approach and demonstrated the most confident inter-concept thinking.

3.1.4 Indicator 3: Real-World Connections

No subject fulfilled Indicator 3 on the written test. Interview exploration revealed a clear divide in how subjects experienced contextual problems, which was closely tied to their prerequisite foundation. For S3, S8, and S9, and to a lesser extent S2, the contextual framing of the problem was perceived as helpful. S9 was able to draw the drone scenario from the problem description alone. Meanhile, S3 demonstrated the ability to construct a mathematical model from contextual information.

R : "How did you connect the contextual word problem to a mathematical model?"
S3 : "By creating the equations, Sir. According to what is in the problem. For example, there is a central post, then we draw the picture, P is at coordinate 0,0, and then just plug it in."
R : "In that case, does a problem with context like this help or confuse you?"
S3 : "It actually helps, Sir. It helps a lot. It is just that sometimes I do not know what the next stage is."

S2 echoed this sentiment, noting that contextual information provided useful additional data.

R : "For you personally, does a real-world context problem actually confuse or help you?"
S2 : "It actually helps, Sir. Because a lot is given, so there is additional information. If it is just a picture without a story, it is lacking."

For S1, S4, S5, and S6, the contextual presentation added cognitive burden rather than providing support. S4 described the double difficulty of word problems.

R : "In your opinion, what is the cause of that difficulty?"
S4 : "I simply do not like mathematics, Sir. So I also lack practice. Furthermore, in my opinion, word problems are doubly difficult; you have to read first, then calculate."

S6 preferred direct mathematical presentation over contextual narrative.

R : "Why are they confusing?"

S6 : "I need the direct ones, for example, find what the angle is, just like that. With something like this, I end up reading it over and over until I am confused from beginning to end."

S1 encountered an additional barrier at the level of mathematical vocabulary.

R : "In terms of the language, did you have any difficulties, for example, terms that you did not quite understand?"
S1 : "Yes, Sir. I did not really understand the terms either."
R : "Does a problem with a daily life context like this, in your opinion, help you form a picture or rather confuse you?"
S1 : "It confuses me, Sir. Because I do not really like mathematics, everything becomes confusing."

S7 occupied a middle ground, contextual problems with a single concept were manageable, but multi-concept contextual problems created confusion.

R : "Do contextual problems or word problems, in your opinion, help provide direction or instead confuse you?"
S7 : "If the word problem does not have many concepts, I can still do it. But if the word problem has multiple concepts, then I get confused. As long as it is a single concept, like Pythagoras made into a story about area, I can still manage."

3.1.5 Connection Difficulty Levels Identified Through Interviews

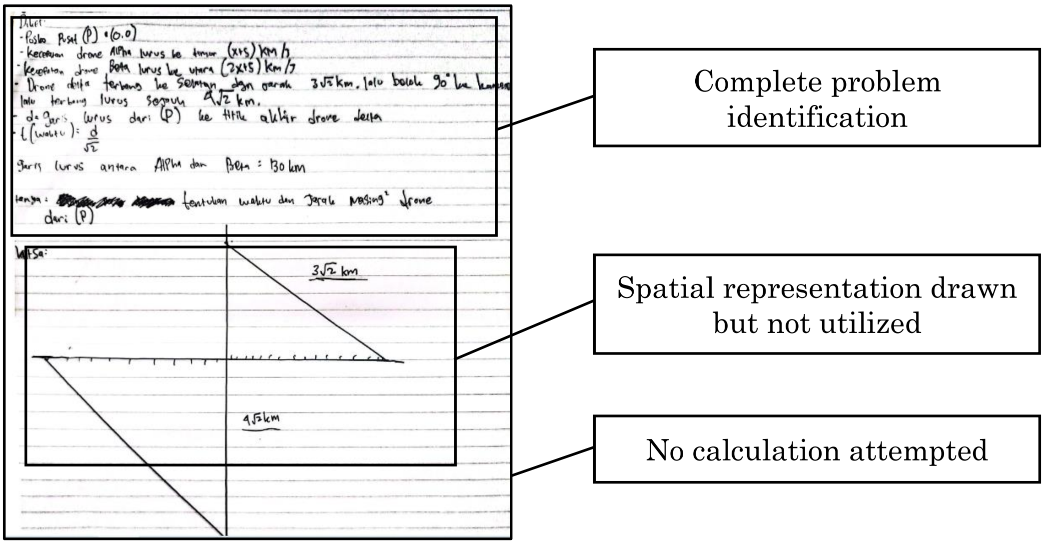
Cross-subject interview analysis identified three qualitatively distinct levels of connection difficulty. These levels describe the nature of each subject's barrier and are independent of the indicator-fulfillment classifications determined by the written test. The distribution is presented in Table 2.

Table 2. Distribution of Interview-Based Connection Difficulty Levels

Level	Description	Subjects
Level A	Weak prerequisite mastery	S1, S4, S5, S6
Level B	Isolated understanding without integration	S2, S7, S10
Level C	Incomplete execution despite connection awareness	S3, S8, S9

The qualitative differences across the three levels are further illustrated by examining subjects' written work, presented in Figures 1–3.

Level A subjects (S1, S4, S5, S6) lacked sufficient prerequisite knowledge to begin meaningful connection at any indicator. Their thinking did not progress to the point where any inter-concept relationship could be formed. The causes of this prerequisite gap varied: extended absences from school due to sports (S6), teacher absenteeism during middle school (S1), motivational disengagement from mathematics (S4), and persistent learning difficulties despite sustained effort (S5). What these subjects shared was that their functional knowledge of the three prerequisite concepts, linear equations, distance-speed-time, and the Pythagorean theorem, was either absent or too fragmentary to serve as a foundation. As shown in Figure 1, S5 transcribed the problem information completely and drew a spatial diagram, yet the answer section remained entirely empty with no mathematical operation initiated.



Diket:

- Posko Pusat (P) = (0.0)
- kecepatan drone Alpha lurus ke timur (x+5) km/j
- kecepatan drone Beta lurus ke utara (2x+5) km/j
- Drone delta terbang ke Selatan dgn jarak 3√2 km. lalu belok 90° ke kanan lalu terbang lurus sejauh 4√2 km.
- d = garis lurus dari (P) ke titik akhir drone delta
- t (waktu) = $\frac{d}{\sqrt{2}}$

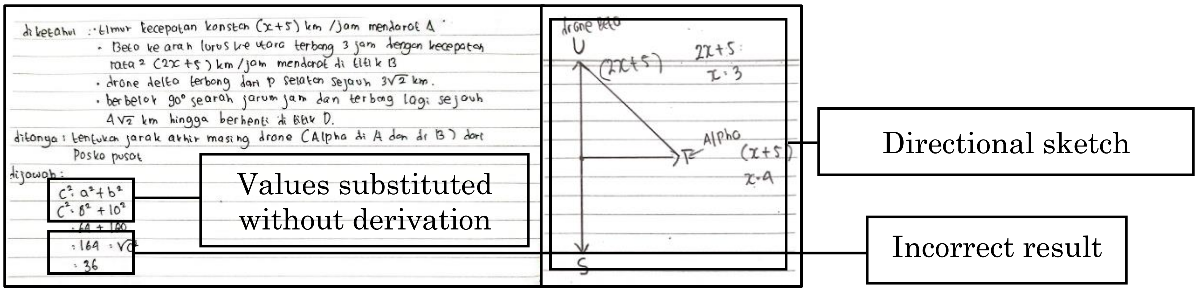
Garis lurus antara Alpha dan Beta = 30 km

tanya: tentukan waktu dan jarak masing² drone dari (P)

Jawab:

Figure 1. Written work of S5 (Level A)

Level B subjects (S2, S7, S10) possessed functional understanding of individual concepts but could not integrate them into a coherent solution path. Their barrier was not knowledge absence but knowledge fragmentation: each concept existed as an independent schema without connections to others. As shown in Figure 2, S7 drew a correct directional sketch and applied the Pythagorean theorem formula. However, values were substituted into the Pythagorean formula without prior derivation from the problem data, and the preceding steps needed to determine the correct distances were entirely absent.



diketahui : • timur kecepatan konstan (x+5) km/jam mendarat A

- Beta ke arah lurus ke utara terbang 3 jam dengan kecepatan rata² (2x+5) km/jam mendarat di titik B
- drone delta terbang dari P selatan sejauh 3√2 km.
- berbelok 90° searah jarum jam dan terbang lagi sejauh 4√2 km hingga berhenti di titik D.

ditanya : tentukan jarak akhir masing drone (Alpha di A dan di B) dari Posko pusat

dijawab :

$c^2 = a^2 + b^2$

$c^2 = 8^2 + 10^2$

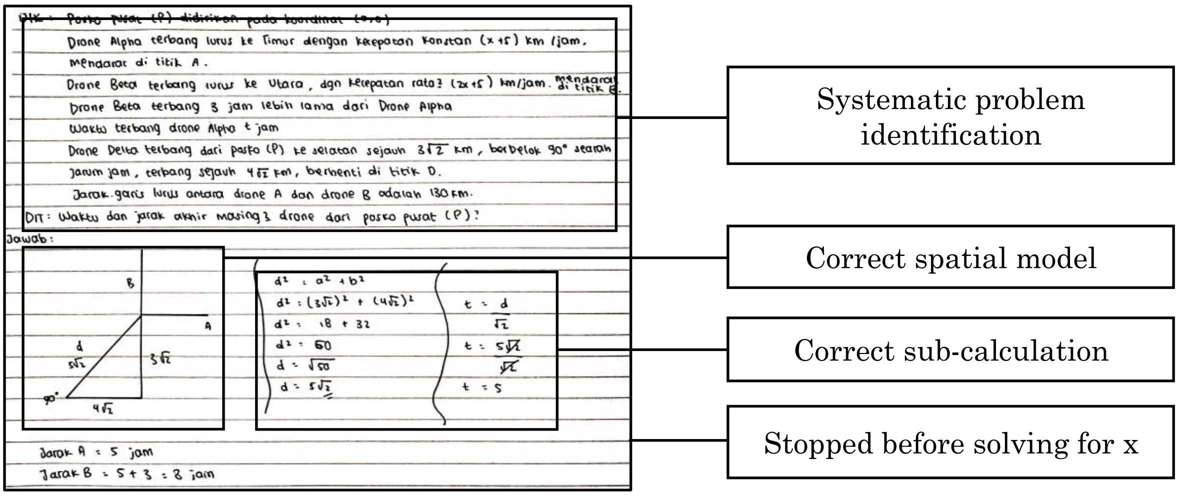
$= 64 + 100$

$= 164 = \sqrt{c^2}$

$= 36$

Figure 2. Written work of S7 (Level B)

Level C subjects (S3, S8, S9) demonstrated the most sophisticated thinking among all subjects. They identified multiple relevant concepts and showed awareness of how those concepts might relate. Their barriers were more specific: S8 did not recognize the need to solve the linear equation before computing distances; S9 had the conceptual capacity but was prevented from applying it by test anxiety; and S3 had the strongest overall foundation but was constrained by the novelty of such a complex integrated problem under test conditions. As shown in Figure 3, S8 set up the problem systematically, constructed an accurate directional diagram, and correctly computed the Delta drone distance. However, the work terminated at the point where the linear equation needed to be solved to determine the speed variable.



Dik : Posko pusat (P) didirikan pada koordinat (0,0)
Drone Alpha terbang lurus ke Timur dengan kecepatan konstan $(x+5)$ km/jam, mendarat di titik A.
Drone Beta terbang lurus ke Utara, dgn kecepatan rata² $(2x+5)$ km/jam. mendarat di titik B.
Drone Beta terbang 3 jam lebih lama dari Drone Alpha
Waktu terbang drone Alpha t jam
Drone Delta terbang dari posko (P) ke selatan sejauh $3\sqrt{2}$ km, berbelok 90° searah jarum jam, terbang sejauh $4\sqrt{2}$ km, berhenti di titik D.
Jarak garis lurus antara drone A dan drone B adalah 130 km.
Dit : Waktu dan jarak akhir masing² drone dari posko pusat (P)?
Jawab :

$d^2 = a^2 + b^2$
$d^2 = (3\sqrt{2})^2 + (4\sqrt{2})^2$
$d^2 = 18 + 32$
$d^2 = 50$
$d = \sqrt{50}$
$d = 5\sqrt{2}$

$t = \frac{d}{\sqrt{2}}$
$t = \frac{5\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$
$t = 5$

Jarak A = 5 jam
Jarak B = 5 + 3 = 8 jam

Systematic problem identification

Correct spatial model

Correct sub-calculation

Stopped before solving for x

Figure 3. Written work of S8 (Level C)

3.2 Discussion

3.2.1 Mathematical Connection Profile of Low-Achieving Students

The three-level framework of connection difficulty identified in this study extends the work of García-García & Dolores-Flores (2021) by providing a finer-grained characterization of how connection failures occur across distinct stages of the connection process. Previous research has often treated low mathematical connection performance as a uniform outcome; the present findings demonstrate that the barrier operates at qualitatively different points. Level A subjects cannot begin connection because prerequisite schemas are absent or too fragmented. Level B subjects possess the prerequisite knowledge but lack the meta-conceptual awareness needed to sequence and integrate it. Level C subjects have both the knowledge and the awareness but are prevented from full execution by specific cognitive gaps or affective interference.

The finding that Pythagorean theorem recognition was the most spontaneous and widespread among Indicator 1 subjects is consistent with visual representation theory (Presmeg, 2006, 2020). The geometric configuration of direction-based problems activates visual-spatial schemas readily, making Pythagorean theorem recognition more accessible than the recognition of abstract relational concepts such as linear equations. This aligns with Komalasari & Imami (2022), who found that Pythagorean theorem recognition was relatively easier than its application in connection-demanding tasks. The present study adds a qualitative dimension: even when subjects correctly identified the Pythagorean theorem, the nature of that identification varied from purely lexical (S1, S4) to structurally grounded (S3, S8, S9), and this qualitative difference in Indicator 1 directly predicted the depth of connection attempted in Indicators 2 and 3.

For Indicator 2, the specific missing link identified by multiple Level B subjects, namely the need to solve the linear equation before computing distances that points to a gap in what Bingölbali & Coskun (2016) describe as connection sequencing: the understanding of how mathematical concepts build upon one another in a specific order to form a coherent solution. This type of knowledge is not developed through single-concept practice exercises, which dominated the learning histories of nearly all subjects in this study.

The role of contextual problems in Indicator 3 showed a dependency on prerequisite foundations that has direct implications for instructional design. For Level C subjects, the real-world context functioned as a meaningful representational bridge: S9 could draw the drone scenario from descriptive text alone, and S3 used the directional context to organize her equation setup. For Level A subjects, however, the same context added extraneous cognitive load (Sweller, 2024) rather than providing support. This dichotomy extends Hasanah & Aini (2021) by specifying the conditions under which contextual problems help versus hinder: when prerequisite schemas are inadequate, context becomes an additional processing demand rather than a scaffold. The implication is that contextual problem design must be calibrated to students' existing prerequisite foundation, rather than assumed to be universally beneficial.

To provide a comprehensive overview of the three-level framework, Table 3 synthesizes the key characteristics, representative interview quotes, and targeted intervention recommendations for each difficulty level.

Table 3. Distribution of Interview-Based Connection Difficulty Levels

Level	Characteristics	Representative Quote	Intervention Recommendation
Level A	Weak prerequisite mastery. Prerequisite concepts (linear equations, distance-speed-time, Pythagorean theorem) are absent or too fragmentary. Thinking does not progress beyond surface-level recognition. Causes include: school absences due to sports (S6), teacher absenteeism (S1),	*S4: "I do not necessarily understand even a single concept, let alone when they have to be combined."* *S5: "Why does mathematics have to use letters? Why not just use numbers?"*	Prerequisite remediation must precede any connection-building activity. Explicit re-teaching of foundational concepts (variables, basic equations, distance formula) using concrete

	motivational disengagement (S4), and fundamental gaps in algebraic representation (S5).		representations. Gradual transition from arithmetic to algebraic thinking.
Level B	Isolated understanding without integration. Individual concepts are understood but exist as independent schemas without connections to others. Cannot determine the sequencing of concepts for integrated problem-solving. Memorization-based learning produces surface knowledge that fails to transfer to novel formats.	*S10: "I know each of the formulas. But I do not know the order to use them. Where to start, which one to use first, that is what I cannot do."* *S7: "I know the area formulas. But how to get from Pythagoras to the area, that is what I do not understand."*	Explicit instruction on concept sequencing and integration logic. Scaffolded multi-step problems that gradually increase the number of concepts to be connected. Emphasis on understanding why formulas are used, not just how.
Level C	Incomplete execution despite connection awareness. Multiple relevant concepts are identified with awareness of their interrelationships. Barriers are specific: missing a key integration step (S8), test anxiety preventing execution (S9), or problem-format novelty constraining performance (S3). Performance gap between written test and interview is documented.	*S9: "If it is each concept individually, I can do it, but when asked to combine them, I am not sure about the sequence of steps. I am afraid that if the sequence is wrong, the entire answer becomes wrong."* *S8: "I understand the basic formulas. How to link them is what I am still confused about. This is the first time I have encountered such a highly complex problem."*	Graduated multi-concept problem exposure under reduced time pressure. Affective support to address test anxiety and build mathematical confidence. Multi-modal assessment (written + oral) to capture true connection ability. Practice with varied integrated problem formats.

3.2.2 Contributing Factors and Implications

Cross-subject analysis identified five primary contributing factors to the observed connection difficulties. First, prerequisite knowledge gaps from middle school were universal across all ten subjects, though their causes varied. For S6, extended absences due to sports activities produced large content gaps. For S1, frequent teacher absenteeism during middle school left similar gaps. For S4 and S5, motivational disengagement from mathematics led to minimal effort and retention. For S10, the issue was not absence of instruction but the nature of that instruction: years of formula-focused, rote-oriented teaching produced surface knowledge that could not transfer. These varied causes suggest that prerequisite deficits require individualized diagnostic identification rather than a single remedial approach.

Second, the prevalence of rote memorization as a learning strategy was pronounced and self-acknowledged. S10 most explicitly described this, noting that he knows formulas but does not understand why they are used. This pattern is consistent with Komalasari & Imami (2022) and Liwalidya et al. (2024), who identified memorization-based learning as a persistent obstacle to flexible mathematical thinking. The present data extends these findings by showing that even subjects with relatively strong foundations, such as S8 and S9, had internalized individual formulas without developing the inter-formula relational knowledge needed for integrated problem-solving.

Third, insufficient exposure to multi-concept integrated problems was noted by S9 and S10 explicitly and was evident in the responses of nearly all subjects. Most described their prior practice as consisting of single-concept exercises, which meant that encountering a problem requiring simultaneous use of linear equations, distance-speed-time, and the Pythagorean theorem was genuinely novel. This supports recommendations by Aziz et al. (2025) for graduated problem complexity in mathematics instruction.

Fourth, affective factors manifested differently across subjects but were influential across all three difficulty levels. Motivational withdrawal in S4 and S5 reduced engagement with mathematics from early schooling. Low self-efficacy in S1 and S2 shaped their initial responses to the problem before any mathematical processing began. Test anxiety in S9 produced a documented gap between her demonstrated competence in interview conditions and her written test performance. These findings are consistent with Hidayah et al. (2024), who identified affective variables as significant predictors of connection performance.

Fifth, near-universal dependence on passive learning resources, particularly YouTube tutorial videos, characterized the learning habits of S1, S4, S5, S6, and S10. While such resources can convey procedural steps effectively, they do not require the active conceptual exploration needed to build inter-concept connections. S3 stands as a contrasting case: her relatively stronger connection profile is consistent with her history of receiving intensive, mathematics-specific tutoring with high-volume varied practice twice weekly throughout middle school.

4. CONCLUSION

Based on the preliminary written test, all ten subjects fulfilled at most one NCTM mathematical connection indicator. Nine subjects fulfilled only Indicator 1 with scores ranging from 2 to 5 out of 6, one subject fulfilled only Indicator 2, and no subject fulfilled Indicator 3. Interview exploration revealed that within Indicator 1, subjects' thinking ranged from purely lexical concept recognition to structured multi-concept identification.

For Indicator 2, thinking ranged from complete inability to begin connection to partial awareness of integration pathways that could not be fully executed. For Indicator 3, the role of contextual framing was contingent on prerequisite foundation: it served as a scaffold for stronger subjects and as an additional burden for weaker ones.

Three qualitative levels of connection difficulty were identified: Level A (weak prerequisite mastery), Level B (isolated understanding without integration), and Level C (connection awareness without full execution ability). Five contributing factors were identified: prerequisite knowledge gaps, rote memorization, insufficient multi-concept practice, negative affective dispositions, and dependence on passive learning resources. These findings imply that effective instruction for low-achieving students must address each difficulty level differently through prerequisite remediation (Level A), explicit concept sequencing instruction (Level B), and graduated multi-concept exposure with affective support (Level C).

5. RECOMMENDATION

Future research should investigate mathematical connection profiles across broader ability levels and different mathematical topics to examine whether the three-level framework identified here generalizes beyond the Pythagorean theorem context. Intervention studies examining scaffolded instruction, graduated multi-concept tasks, or targeted prerequisite remediation for each difficulty level would translate these descriptive findings into practical pedagogical improvements. A notable limitation of this study is that findings are drawn from a single school context; replication across diverse settings would strengthen the generalizability of the proposed framework.

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