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Optimization of Coffee Inventory and Replenishment Planning under Demand Uncertainty: A Linear Programming Approach

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Abstract

This study develops a multi-period linear programming model to optimize coffee inventory and replenishment planning under demand uncertainty. The model integrates inventory balance, replenishment capacity, storage capacity, and safety stock constraints to determine cost-efficient replenishment quantities and ending inventory levels for three coffee products: Robusta, Arabica, and Blend. Simulated data over six planning periods were analyzed under low, medium, and high demand scenarios using PuLP in Python. The results show that optimal solutions were obtained under low and medium demand conditions, with total inventory costs of Rp 286,836,000 and Rp 480,466,000, respectively. Under low demand, inventory was maintained exactly at safety stock levels, reflecting a just-in-time strategy. Under medium demand, the model temporarily increased Robusta inventory to anticipate future demand. However, the high-demand scenario was infeasible, indicating insufficient replenishment capacity. The model provides a practical decision support tool for cost-efficient and resilient coffee inventory management.

Keywords: coffee inventory; demand uncertainty; linear programming; replenishment planning

Abstrak

Penelitian ini mengembangkan model pemrograman linear multi-periode untuk mengoptimalkan persediaan dan perencanaan pengisian kembali produk kopi dalam kondisi ketidakpastian permintaan. Model mengintegrasikan kendala keseimbangan persediaan, kapasitas pengisian kembali, kapasitas penyimpanan, dan safety stock untuk menentukan jumlah replenishment serta persediaan akhir yang efisien secara biaya pada tiga produk kopi, yaitu Robusta, Arabica, dan Blend. Data simulasi selama enam periode dianalisis pada skenario permintaan rendah, sedang, dan tinggi menggunakan PuLP di Python. Hasil optimasi menunjukkan bahwa solusi optimal diperoleh pada skenario permintaan rendah dan sedang, dengan total biaya persediaan masing-masing sebesar Rp 286.836.000 dan Rp 480.466.000. Pada permintaan rendah, persediaan dipertahankan tepat pada tingkat safety stock, sedangkan pada permintaan sedang model meningkatkan sementara persediaan Robusta untuk mengantisipasi permintaan berikutnya. Skenario permintaan tinggi tidak layak, sehingga diperlukan peningkatan kapasitas dan fleksibilitas persediaan.

Kata Kunci: ketidakpastian permintaan; pemrograman linear; perencanaan replenishment; persediaan kopi

1. INTRODUCTION

Efficient inventory management is an important component of food supply chain planning, especially for coffee based enterprises that must balance product availability, storage limitations, and changing market demand (Ko et al., 2020; Torabzadeh et al., 2022). In coffee businesses, insufficient inventory may lead to unmet demand and potential revenue loss, whereas excessive inventory may increase holding costs and reduce operational efficiency (Alifadillah & Supriatna, 2023).

Coffee inventory planning becomes more challenging when demand fluctuates across periods (Nasution et al., 2025). Demand uncertainty requires enterprises to determine appropriate replenishment quantities while maintaining sufficient stock levels for each product (Kusomrosananan & Phumchusri, 2024). Therefore, a systematic decision-making approach is needed to support inventory and replenishment planning under different demand conditions.

Linear programming provides a mathematical framework for optimizing inventory decisions subject to operational constraints (Kozlova et al., 2025). Through linear programming, decision makers can determine optimal replenishment quantities and inventory levels while considering demand, storage capacity, replenishment limits, holding costs, and purchasing costs (Goyal & Kumar, 2025; Gupta et al., 2023).

Although inventory optimization has been widely studied in supply chain management, its application in coffee based food enterprises under multiple demand scenarios remains relevant for further investigation (Kusuma Wardana et al., 2025). Most inventory models focus on minimizing operational costs under a fixed demand setting, whereas real food enterprises often face fluctuating demand across planning periods (Kusomrosananan & Phumchusri, 2024; Pathak et al., 2024). Therefore, scenario based inventory optimization is needed to evaluate how replenishment and inventory decisions change under different demand conditions.

This study develops a linear programming model to optimize coffee inventory and replenishment planning under demand uncertainty. The proposed model considers three coffee products and six planning periods. The contribution of this study lies in the formulation of a scenario-based linear programming model that integrates inventory balance, replenishment planning, storage capacity, and safety stock constraints within a unified mathematical framework.

2. MATERIALS AND METHODS

Research Design

This study employs a quantitative mathematical modeling approach using linear programming to optimize inventory and replenishment planning for coffee products under demand uncertainty (Islamiyah et al., 2025). The model considers multiple planning periods and evaluates inventory decisions under three demand scenarios: low demand, medium demand, and high demand.

The objective of the model is to determine the optimal replenishment quantity and ending inventory level for each coffee product in each planning period. The optimization model minimizes total inventory-related cost consisting of replenishment cost and inventory holding cost (Nasution et al., 2025).

The model uses simulated data representing the operational characteristics of a hypothetical coffee-based enterprise. The use of simulated data is intended to evaluate the mathematical behavior of the proposed model rather than to describe the empirical condition of a specific company (Bolaños-Zúñiga & Vidal-Holguin, 2020).

Research Framework

The research procedure consists of several stages:

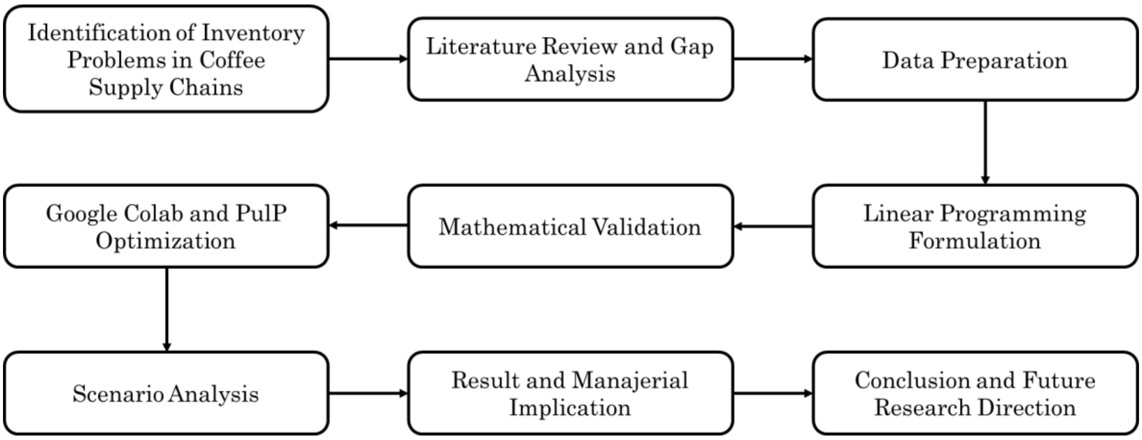

Figure 1. Research Framework

The framework begins with the preparation of inventory parameters and demand scenarios. Subsequently, a linear programming model is formulated to represent the inventory balance, replenishment limits, storage capacity, and safety stock requirements. The model is then solved for each demand scenario to obtain the optimal replenishment and inventory decisions.

Set and Indices

Table 1. Set and Indices

Symbol	Description
i	Coffee product index
t	Planning period index
$I = \{1,2,3\}$	Set of coffee products
$T = \{1,2,3,4,5,6\}$	Set of planning periods

Table 2. Coffee Product Codes

Index	Product
1	Robusta
2	Arabica
3	Blend

Table 3. Planning Periods

Index	Period
1	Month 1
2	Month 2
3	Month 3
4	Month 4
5	Month 5
6	Month 6

Model Parameter

Table 4. Model Parameter

Parameter	Description
D_{it}	Demand of product i in period t
c_i	Replenishment cost per kilogram of product i
h_i	Holding cost per kilogram of product i per period
S_i	Maximum storage capacity of product i
R_i^{max}	Maximum replenishment quantity of product i per period
SS_i	Safety stock requirement of product i
I_{i0}	Initial inventory of product i

Decision Variable

The decision variables used in this model are:

$$R_{it} \ \ and \ \ I_{it}$$

where R_{it} is replenishment quantity of product i in period t and I_{it} is ending inventory level of product i in period t.

These decision variables determine how much coffee product should be replenished and how much inventory should remain at the end of each planning period.

Objective Function

The objective of the model is to minimize total inventory-related cost over the planning horizon.

$$\min Z = \sum_{i \epsilon I} \sum_{t \epsilon T} (C_i R_{it} + h_i I_{it})$$

where

Z = total inventory management cost

$C_i R_{it}$ = replenishment cost of product i in period t

$h_i I_{it}$ = holding cost of product i in period t

The objective function minimizes the combined cost of replenishment and inventory holding. This formulation enables the enterprise to maintain product availability while avoiding excessive inventory accumulation.

Constraints

Inventory Balance Constraint

The ending inventory of each product in each period is determined by the previous inventory, replenishment quantity, and demand.

$$I_{it} = I_{i,t-1} + R_{it} - D_{it}$$

for

$$i \in I, \qquad t \in T$$

For the first period, the inventory balance uses the initial inventory:

$$I_{i1} = I_{i0} + R_{i1} - D_{i1}$$

where I_{i0} is initial inventory of product i. This constraint represents the dynamic flow of inventory over the planning horizon.

Storage Capacity Constraint

Inventory level must not exceed the available storage capacity.

$$I_{it} \leq S_i$$

for

$$i \in I, \qquad t \in T$$

where S_i is maximum storage capacity of product i.

Replenishment Capacity Contsraint

$$R_{it} \leq R_i^{max}$$

for

$$i \in I, \qquad t \in T$$

where R_i^{max} is maximum replenishment capacity of product i.

Safety Stock Constraint

$$I_{it} \geq SS_i$$

for

$$i \in I, \qquad t \in T$$

where SS_i is safety stock requirement of product i. Safety stock is included to reduce the risk of stockout under uncertain demand conditions.

Non-Negativity Constraints

All decision variables must be non-negative.

$$R_{it} \geq 0$$

$$I_{it} \geq 0$$

for

$$i \in I, \qquad t \in T$$

Complete Mathematical Model

The complete linear programming model is formulated as follows:

$$\min Z = \sum_{i \epsilon I} \sum_{t \epsilon T} (C_i R_{it} + h_i I_{it})$$

Subject to:

$$I_{it} = I_{i,t-1} + R_{it} - D_{it}$$

$$I_{it} \leq S_i$$

$$R_{it} \leq R_i^{max}$$

$$I_{it} \geq S_i$$

$$R_{it} \geq 0$$

$$I_{it} \geq 0$$

for all

$$i \in I, \qquad t \in T$$

Mathematical Model Validation

Feasibility

A linear programming model is feasible if there exists at least one solution that satisfies all constraints simultaneously (Woubante, 2017). In the proposed model, feasibility is ensured by allowing replenishment decisions to satisfy demand while maintaining the required safety stock level, subject to replenishment and storage capacities.

Boundedness

The model is bounded because the inventory and replenishment variables are limited by finite upper bounds (Achkar et al., 2024). The storage capacity constraint is:

$$I_{it} \leq S_i$$

and the replenishment capacity constraint is:

$$R_{it} \leq R_i^{max}$$

Since all decision variables are bounded by finite constants, the objective function cannot decrease indefinitely. Therefore, the model has a finite optimal solution.

Linearity

The objective function

$$\min Z = \sum_{i \epsilon I} \sum_{t \epsilon T} (C_i R_{it} + h_i I_{it})$$

is linear because all decision variables appear with degree one and all coefficients are constants (Aouam et al., 2021). The constraints are also linear because they consist only of linear combinations of the decision variables. Therefore, the model satisfies the assumptions of linear programming.

Computational Validation

The model will be implemented using the PuLP optimization package in Python. An optimal solution status returned by the solver indicates that the model is computationally feasible and numerically stable for the simulated demand scenarios (Paradis, 2025).

Simulated Data

Table 5. Initial Inventory

Product	Initial Inventory
Robusta	300
Arabica	250
Blend	200

Table 6. Replenishment Cost

Product	Replenishment Cost (Rp/kg)
Robusta	55000
Arabica	78000
Blend	65000

Table 7. Holding Cost

Product	Holding Cost (Rp/kg/period)
Robusta	1200
Arabica	1800
Blend	1500

Table 8. Storage Capacity

Product	Storage Capacity (Kg)
Robusta	1000
Arabica	800
Blend	900

Table 9. Maximum Replenishment Capacity

Product	Maximum Replenishment (Kg/period)
Robusta	500
Arabica	500
Blend	600

Table 10. Safety Stock

Product	Safety Stock (Kg)
Robusta	120
Arabica	100
Blend	150

Table 11. Low Demand Scenario

Period	Robusta	Arabica	Blend
1	250	200	180
2	280	220	200
3	300	240	220
4	320	260	240
5	340	280	260
6	360	300	280

Table 12. Medium Demand Scenario

Period	Robusta	Arabica	Blend
1	350	300	250
2	400	350	300
3	450	400	350
4	500	450	400
5	550	500	450
6	600	550	500

Table 13. High Demand Scenario

Period	Robusta	Arabica	Blend
1	450	400	350
2	500	450	400
3	550	500	450
4	600	550	500
5	650	600	550
6	700	650	600

3. RESULTS AND DISCUSSIONS

Total Inventory Cost under Different Demand Scenarios

The optimization model was first evaluated under three demand scenarios, namely low, medium, and high demand conditions. Table 14 presents the optimization status and total inventory cost obtained from the linear programming model.

Table 14. Optimization Results under Different Demand Scenarios

Scenario	Optimization Status	Total Inventory Cost (Rp)
Low Demand	Optimal	286,836,000
Medium Demand	Optimal	480,466,000
High Demand	Infeasible	602,086,000

The results indicate that the proposed inventory system successfully generated optimal solutions under low and medium demand scenarios. Under low demand conditions, the minimum total inventory cost was Rp 286,836,000. When demand increased to the medium level, the total inventory cost rose substantially to Rp 480,466,000. The percentage increase in inventory cost from low to medium demand is 67.5%.

This finding suggests that increasing demand substantially affects inventory-related expenditures, primarily due to higher replenishment activities required to maintain service continuity and safety stock requirements.

In contrast, the high-demand scenario resulted in an infeasible solution. This outcome indicates that the existing replenishment capacities and safety stock policies were insufficient to accommodate extreme demand conditions. Therefore, the current inventory system can adequately support normal and moderate market conditions but may require capacity expansion or alternative replenishment strategies during demand surges.

Replenishment Policy under Low Demand Conditions

Table 15 presents the optimal replenishment quantities under low demand conditions.

Table 15. Optimal Replenishment Plan under Low Demand

Period	Robusta	Arabica	Blend
1	70	80	100
2	220	200	280
3	240	220	300
4	260	240	320
5	280	260	340
6	300	280	360

The replenishment quantities gradually increased across the planning horizon following the growth in demand. The optimization model determined replenishment levels sufficient to satisfy demand while avoiding excessive inventory accumulation. Table 16 reports the corresponding ending inventory levels.

Table 16. Ending Inventory Levels under Low Demand

Period	Robusta	Arabica	Blend
1	120	100	150
2	120	100	150
3	120	100	150
4	120	100	150
5	120	100	150
6	120	100	150

Interestingly, inventory levels remained constant throughout the six planning periods. The ending inventories exactly matched the predefined safety stock levels for each product. Specifically,

$$I_{Arabica} = 100 \text{ kg}, \qquad I_{Blend} = 150 \text{ kg}, \qquad I_{Robusta} = 120 \text{ kg}$$

This result demonstrates that the model adopted a just-in-time inventory strategy, where replenishment decisions were made only to maintain minimum safety stock requirements while minimizing holding costs.

Replenishment Policy under Medium Demand Conditions

As demand increased, the replenishment strategy changed substantially. Table 17 presents the optimal replenishment quantities under medium demand conditions.

Table 17. Optimal Replenishment Plan under Medium Demand

Period	Robusta	Arabica	Blend
1	170	150	200
2	350	300	400
3	400	350	450
4	500	400	500
5	500	450	550
6	500	500	600

The replenishment quantities increased steadily over time. Notably, Robusta reached its maximum replenishment capacity of 500 kg during the final period, while Arabica attained its maximum replenishment limit of 500 kg beginning in Period 4. These observations indicate that the system gradually approached its operational limits as demand intensified. Table 18 shows the corresponding inventory levels.

Table 18. Ending Inventory Levels under Medium Demand

Period	Robusta	Arabica	Blend
1	120	100	150
2	120	100	150
3	120	100	150
4	170	100	150
5	170	100	150
6	120	100	150

Most products maintained inventories equal to their safety stock levels. However, Robusta inventory temporarily increased from its safety stock level of 120 kg to 170 kg during Periods 4 and 5. This additional inventory suggests that the optimization model strategically accumulated inventory to anticipate future demand while respecting replenishment capacity constraints. This proactive behavior reflects the adaptive capability of the proposed inventory model under moderate demand pressure.

Inventory Dynamics and Manajerial Implication

Figure 2 illustrates the total inventory costs under feasible demand scenarios.

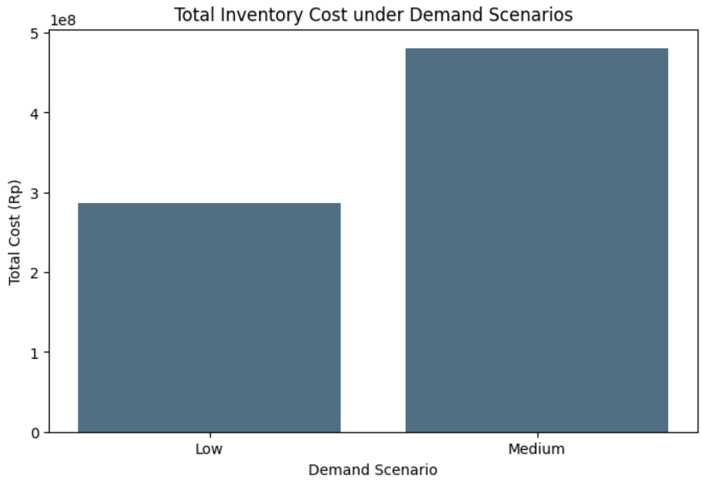

Figure 2. Total Inventory Cost under Demand Scenarios

The figure confirms that medium demand conditions require substantially higher inventory expenditures than low demand conditions. The sharp increase in total cost emphasizes the sensitivity of inventory systems to demand growth.

Figure 3 depicts the inventory dynamics under medium demand conditions.

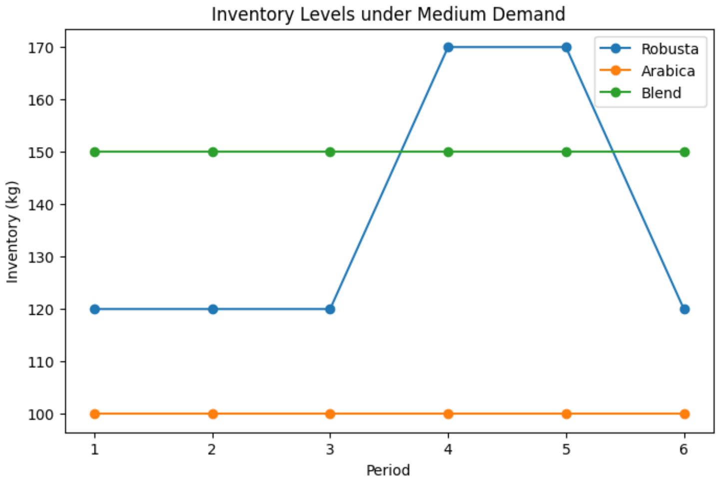

Figure 3. Inventory Levels under Medium Demand

The inventory trajectories reveal that Arabica and Blend consistently operated at their safety stock levels, whereas Robusta exhibited temporary inventory buildup during Periods 4 and 5. Such behavior indicates the model's ability to balance holding costs against replenishment limitations.

Figure 4 presents the replenishment trajectories under medium demand conditions.

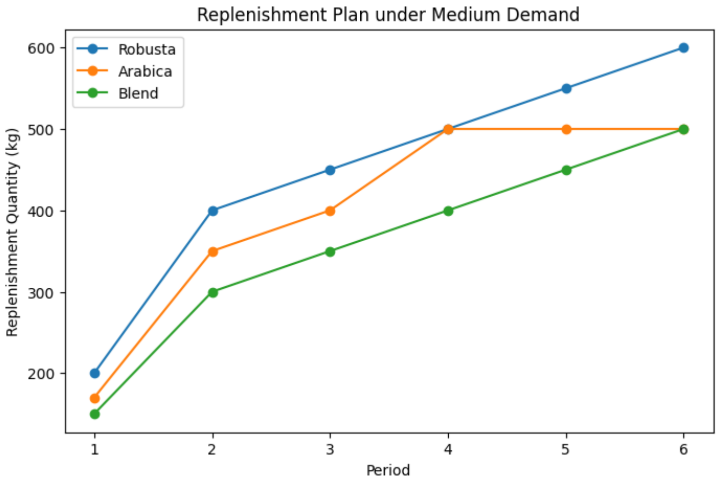

Figure 4. Replenishment Plan under Medium Demand

The replenishment trends indicate that several products operated near their maximum replenishment capacities. Consequently, the infeasibility observed under the high-demand scenario can be attributed to insufficient replenishment capability relative to demand growth.

From a managerial perspective, these findings imply that the current inventory configuration is adequate under normal and moderate market conditions. However, firms expecting substantial demand increases should consider expanding replenishment capacities, revising safety stock policies, or diversifying supply sources to improve system resilience and prevent inventory shortages (Cotta & Salvador, 2020).

4. CONCLUSION

This study proposed a linear programming model to optimize coffee inventory and replenishment planning under demand uncertainty. The model integrated replenishment decisions, inventory balance constraints, storage capacity limitations, replenishment capacity constraints, and safety stock requirements within a multi period planning framework.

The optimization results demonstrated that the proposed model successfully generated feasible and optimal solutions under low and medium demand scenarios, with total inventory costs of Rp 286,836,000 and Rp 480,466,000, respectively. Under low demand conditions, the model maintained inventory levels exactly at the predefined safety stock levels, reflecting an efficient just in time inventory strategy. Under medium demand conditions, the model exhibited adaptive behavior by temporarily increasing Robusta inventory to anticipate future demand while operating near replenishment capacity limits.

In contrast, the high-demand scenario resulted in an infeasible solution, indicating that the existing replenishment capacities and safety stock policies were insufficient to satisfy substantial increases in demand. This finding highlights the importance of capacity planning and inventory flexibility in maintaining supply chain resilience during demand surges.

Overall, the proposed linear programming approach provides a practical decision support tool for inventory management in coffee businesses. The model can assist managers in determining cost efficient replenishment policies while ensuring service continuity under uncertain demand conditions.

Future studies may extend the proposed framework by incorporating stochastic demand, lead-time uncertainty, perishability considerations, or multi-objective optimization approaches to better represent the complexities of real world coffee supply chains.

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