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Stability analysis of nonlinear dynamic systems using the lyapunov method approach

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Abstract
A new approach to the stability analysis of homogeneous nonlinear systems is proposed, based on the concept of candidate Lyapunovfunctions, where the focus is not on the positive definiteness of the candidate Lyapunovfunctions, but on the negative definiteness of their derivatives. After having ensured the negative definiteness of the derivative function, on the basis of the sign assignment of the primitive function, the stability of the equilibrium is analyzed, where the necessary and sufficient conditions are declared at the same time. The selection of the tendency of the Lyapunov candidate function is primarily performed in the form of a linear combination of some simple functions. The unknown coefficients in the structure of the candidate function are computed based on the negative definiteness of the derivative function. Then, using these coefficients in Lyapunov function, sign of primitive function in state space is argued. Thus, the stability/instability of the equilibrium point can be deduced from the triple sign settings of the candidate function. Furthermore, in the process of negative definiteness of the derivative function, the coefficients are obtained using two independent methods. The proposed theoretical results are supported and their effectiveness is demonstrated by numerical simulations.

Keywords: analysis; lyapunov; nonlinear

Abstrak
Pendekatan baru terhadap analisis stabilitas sistem nonlinier homogen diusulkan, berdasarkan konsep kandidat fungsi Lyapunov, yang fokusnya bukan pada kepastian positif dari kandidat fungsi Lyapunov, namun pada kepastian negatif turunannya. Setelah memastikan kepastian negatif dari fungsi turunan, berdasarkan penetapan tanda fungsi primitif, stabilitas kesetimbangan dianalisis, di mana kondisi perlu dan kondisi cukup dinyatakan pada saat yang bersamaan. Pemilihan kecenderungan fungsi kandidat Lyapunov terutama dilakukan dalam bentuk kombinasi linier beberapa fungsi sederhana. Koefisien yang tidak diketahui dalam struktur fungsi kandidat dihitung berdasarkan kepastian negatif dari fungsi turunan. Kemudian, dengan menggunakan koefisien-koefisien ini dalam fungsi Lyapunov, tanda fungsi primitif dalam ruang keadaan diperdebatkan. Dengan demikian, kestabilan/ketidakstabilan titik kesetimbangan dapat disimpulkan dari pengaturan tanda tripel fungsi kandidat. Selanjutnya pada proses kepastian negatif fungsi turunan, koefisien diperoleh dengan menggunakan dua metode independen. Hasil teoritis yang diusulkan didukung dan efektivitasnya ditunjukkan oleh simulasi numerik.

Kata Kunci: analisis; lyapunov; nonlinier

1. INTRODUCTION

The significance of stability in control systems, whether linear or nonlinear, is widely recognized. Consequently, researchers in the field of control systems have developed appropriate methods for analyzing system stability. As noted by Fawzi et al. (2014), stability analysis is particularly important for cyber-physical systems (CPS), which are vulnerable to numerous attacks. Stability is a critical factor in various industrial systems, including breakwaters, structural reliability analysis, and inverter-based nonlinear power systems, and recent advances in this area should also be considered.

Lyapunov methods are used to analyze the stability of dynamical systems. However, the first Lyapunov method determines only the local stability of an equilibrium point, which makes it impractical for highly nonlinear systems, such as chaotic systems. The second or direct Lyapunov approach provides a sufficient condition for the stability of the equilibrium point, but does not provide a systematic or cumulative approach to finding a candidate function that satisfies the Lyapunov conditions.

Several attempts have been made to develop a method for selecting the Lyapunov function, each with its own advantages and disadvantages. However, the first method of Lyapunov only determines the local stability of an equilibrium point. This makes it impractical for highly nonlinear systems, such as chaotic systems.The article presents a method for forming a Lyapunov function using linear programming for autonomous systems. The process of establishing a Lyapunov function in the square form for polynomial systems of positive dimensions is discussed in Ji Zu, et al. (2013). The method defines a square Lyapunov function with some unknown coefficients, which are calculated using the Homotopy continuation algorithm. A method for determining the Lyapunov function for the desired switching dynamical systems is provided based on Schwartz & Yan (1997). A method for determining the Lyapunov function for nonlinear systems is presented in the form of normal coordinates, using the theory of normal forms.

Additionally, the language should be clear, objective, and value-neutral, avoiding biased, emotional, figurative, or ornamental language. Finally, the content of the improved text must be as close as possible to the source text, and the addition of further aspects must be avoided at all costs.

The stability of the equilibrium point of nonlinear systems can be analyzed using a Lyapunov function. One method for constructing this function is the sum of squares method (SOS), which involves forming the Lyapunov function as a sum of pairwise polynomial expressions raised to powers. This method constructs a positive function that can be used to solve a convex optimization problem and calculate the unknown coefficients. The method has been developed numerically for both continuous and discrete-time systems.

Additionally, presents a general structure for stability analysis of nonlinear systems based on the Sum Of Squared method. This method involves decomposing the vector into another system, which is explained as a polynomial vector field with no memory sentences. It is important to note that all methods for calculating the Lyapunov function are based on a positive function expressed as a sum of squares. Additionally, the derivative of this function provides a symmetric representation of a sum of squares.

We have introduced a new approach to constructing the Lyapunov function, which is fundamentally different from existing methods. Our primary focus was on defining the derivative of a function and then defining the sign of the candidate function itself in the domain space. The stability of the equilibrium point was expressed as a necessary and sufficient condition using this method. This method is distinguished from others by starting with $\dot{V}$ and then reaching V, whereas other methods begin with a V as the candidate for Lyapunov's function and then determine $\dot{V}$ to assess the stability of the equilibrium point. In addition, the paper introduces an innovative approach of providing Lyapunov functions based on a linear combination of simple functions. The coefficients of these functions were computed analytically. This approach differs from the sum of squares method, where the calculation of V and $\dot{V}$ is done simultaneously to ensure the proper symbols of each function.

This paper extends the method of Meigoli et al. (2017) by numerically calculating the coefficients of the Lyapunov function, allowing for the solution of a wider range of problems. The coefficients are determined through the derivation of the aforementioned function. The main focus of this paper is on standard homogeneous nonlinear systems, which are widely used in various branches of science and engineering. We use these systems to form a candidate Lyapunov function in the form of a linear combination of sentences of the same degree. The candidate function structure presented in this article offers a systematic approach to stability analysis, which is considered one of its innovative features.

The method of calculating unknown coefficients in the SOS approach is fundamentally based on convex optimization. However, in this paper, we use two simple algorithms based on Least Squares to calculate the unknown coefficients.

2. RESEARCH METHOD

A quantitative descriptive method was adopted for the investigation. According to Sugiyono (2019), the quantitative technique is a research approach based on positivism or scientific philosophy with concrete and practical scientific principles. Meanwhile, Andriani (2015) defines quantitative approaches as methods that use research data in the form of numbers and also data analysis.

3. RESULT & DISCUSSION

This paper studies a specific type of nonlinear systems known as standard homogeneous nonlinear systems. Homogeneous polynomial differential equations are prevalent in various fields of science and engineering due to their unique characteristics. In general, many terminologies and analytical points in the field of linear systems can be extended to nonlinear homogeneous systems. One unique feature of these systems is the equivalence of many local attributes around the equilibrium point and globality. For instance, if the equilibrium point of a homogeneous nonlinear system is locally stable, it is also globally stable. These and other features have made these systems widely used in modeling and describing physical systems.

According to Definition 1, a polynomial is considered homogeneous if all of its terms have the same degree. In other words, a function $V(X) : Rn \rightarrow R$ is homogeneous of degree p if it satisfies the condition for all $\lambda \in R$.

$V(\lambda x) = \lambda^p V(x)$

In this case, we write V ∈ H

According to Definition 2, a dynamical system x = f(x) is said to be homogeneous of k degree $f(x) = [f_1(x)f_2(x)\ldots\ f_n(x)]^T$ if its vector field satisfies the following condition:

$f(\lambda x) = \lambda^k f(x)$

In this case, we write $f \in S_k$

From Eq. (1), it is evident that if the homogeneous polynomial V has the same sign on an arc of the unit circle {z ∈ Rn : x = 1}, then the function will be the same on the radial part of that arc. This feature is illustrated in Fig. 1, where the yellow sector represents the expansion of a branch of a corresponding arc, in which the polynomial does not change its sign. Therefore, the task of determining the sign of a homogeneous polynomial is reduced to determining its sign on a single circle.

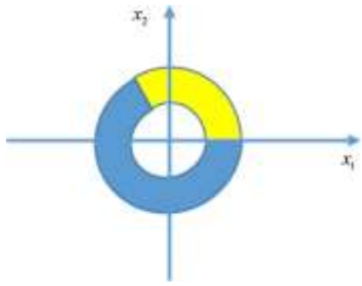

Figure 1. Demonstrating the absence of sign changes in a homogeneous polynomial through a sector expansion.

Definition 3: The p-form representation of a vector x ∈ Rn, where p ∈ ℕ, is defined as follows:

$$x^{[p]} = \left(x_1^p, x_1^{p-1}x_2, x_1^{p-1}x_3, \ldots, x_1^{p_1}x_2^{p_2}, \ldots, x_n^{p_n}, \ldots, x_n^p\right)$$

This p-form representation contains sentences as $x^{p_1}, x^{p_2}, x^{p_n}$ which

$$p_1 + p_2 + \cdots + p_n = p, \quad p_i \in N_0$$

Where $N_0 = N \ \{0\}$.
To create the z[P] representation in a standard way, lexicography can be used. For instance, when p = 5 and n = 3, the lexicographic order is as follows:

$$x^{[5]} = (x_1^5, x_1^4x_2, x_1^4x_3, x_1^3x_2^2, x_1^3x_2x_3, x_1^3x_3^2, x_1^2x_2^3, x_1^2x_2^2x_3, x_1^2x_2x_3^2, x_1^2x_3^3, x_1x_2^4, x_1x_2^3x_3, x_1x_2^2x_3^2, x_1x_2x_3^3, x_1x_3^4, x_2^5, x_2^4x_3, x_2^3x_3^2, x_2^2x_3^3, x_2x_3^4, x_3^5)$$

For any integers p and n, it can be demonstrated that there exist m = These systems involve functions of a single sentence of p degree in a p-form representation. The next section utilizes homogeneous dynamic systems with useful features. It is assumed that the equilibrium point of the system is at the origin.

Lemma 1: Assuming that the dynamic system $\dot{z}$ = (z) is homogeneous. If the vector field is continuous and the equilibrium point of the system is stable, then a homogeneous Lyapunov function can be used to prove the stability of the system. This lemma actually limits the search scope to the Lya- punov function for homogeneous functions, which can be valuable from the computational point of view. The following lemma provides a comprehensive version of this proposition.

Lemma 2 [27]: For the homogeneous dynamical system $\dot{z}$ = (z), which belongs to Sk, the derivative $\dot{V}$ ∈ Hp+k−1 of the function V ∈ Hp+k−1 along the solutions of the system is a homogeneous function of degree p + k − 1.
The Lyapunov theorem (Lyapunov's direct method) is the most well-known theorem for analyzing and designing nonlinear systems. It provides sufficient conditions for the stability of the equilibrium point. Additionally, in [28], the instability theory demonstrates the unstable equilibrium point for unstable systems. This section discusses equilibrium stability by first deriving the Lyapunov candidate function with a negative definite term and then determining the sign of the function.

Theorem 1: Assume the dynamical system $\dot{z}$ = (z). If there exists a continuous partial function V(z) such that V(0) = 0 and $\dot{V}$(z) is negative definite along the system responses, then the equilibrium point of the system is asymptotically stable if V(X) is positive. Otherwise, the equilibrium point is unstable.

We prove that the function V(x) is neither positive nor negative semidefinite for the given dynamical system. Assuming the contrary, if V(x) is positive semi-definite, then at a point x0 outside the origin, we have V(x0) = 0. However, this contradicts the theorem's assumption. The hypothesis is invalidated because, out of origin, V(x) Passing point x0 contradicts the positive semi-definiteness of the V(x) function. characteristic of the V(x) function. Similarly, it can be demonstrated that V(x). never negative semi-definite.

So we may say that V(z) has three states: totally positive definite, totally negative definite, and undetermined. We will examine each of these three modes one by one:

1. If SituationV(x) > 0, then the Lyapunov theorem applies. This theorem states that if there exists a positive function V(x) whose derivative is always negative definite, then the equilibrium point is asymptotically stable.
2. When SituationV(s) is less than zero, the hypotheses of the instability theory can be obtained by setting Λ equal to negative In summary, the dynamical system studied exhibits instability of the equilibrium point due to $\Lambda > 0$ and $\Lambda >\dot{} 0$.
3. V(x) is an indeterminate situation. The change of sign of V(x) around the equilibrium point indicates a negative sign of V(x) in the region surrounding the point. For this situation, the small area adjacent to the equilibrium point has both negative signV(x) and V(x). According to condition 2, we can conclude that the equilibrium point is unstable.

Proof completed. This section focuses on homogeneous systems and proposes two methods for forming the Lyapunov function. The global stability of the equilibrium point is then concluded using these methods. Consider the homogeneous nonlinear dynamical system x = f(x) where f ∈ Sk. The basic functions Vi(x) in the i-th entry of the representation corresponding to state vector x ∈ ℝn are as follows:

$$v_i(\mathbf{x}) = \mathbf{x}_i^{[p]}$$

The function of the Lyapunov candidate is constructed as a linear combination of these basic functions.

$$V(\mathbf{x}) = \sum_{i=1}^{m} a_i v_i(\mathbf{x}) = \mathbf{a}^T(\mathbf{x}^{[p]})^T$$

It called Equation (7) in this research.

$$m = \begin{pmatrix} p+k-1 \\ p \end{pmatrix}$$

The number of terms in p-form is represented by 's', while 'ai' are coefficients that need to be calculated as the process continues. It is important to note that the p-form exhibitions are considered to be linearly independent. Equation (7) has unique linear composition coefficients. However, it is evident that V(0) equals zero. Additionally, based on the symbol presented in Equation (7), the coefficient vector is as follows:

$$\mathbf{a} = \begin{bmatrix} a_1 & a_1 & \cdots & a_m \end{bmatrix}^T$$

Now, by deriving the proposed Lyapunov Equation (7) for the homogeneous system x = f(x) we have:

$$\dot{V}(\mathbf{x}) = \sum_{i=1}^{m} a_i \dot{v}_i(\mathbf{x}) \equiv \mathbf{a}^T H^T (\mathbf{x}^{[p+k-1]})^T < 0$$

It called Equation (8) in this research

The problem involves calculating an intermediate matrix H, which depends on the specific problem. It is important to note that, according to Lemma 2, V˙ (X) is homogeneous of degree p + k − 1, and the linear combination of the base functions Vi (X) in Eq. (9) is also homogeneous of degree p + k − 1. The method for calculating the intermediate matrix is explained in the following section. Thus, the task is to compute a vector that satisfies Eq. (9). To accomplish this, we set V˙(x) equal to a specific negative function and calculate the unknown coefficients. To illustrate general methods, let us first explain the subject with an example. To do this, let us consider the following dynamic system with an equilibrium point at the origin.

$$\begin{cases} \dot{x}_1 = -x_1^3 - 2x_2^3 \\ \dot{x}_2 = 3x_1^3 - 3x_2^3 \end{cases}$$

The following lines analyze the stability status of the equilibrium point. It is obvious that the aforementioned system is homogeneous. Therefore, we can select the Lyapunov function to assess stability. The function is a homogeneous form of degree 4.

$$V(\mathbf{x}) = a_1x_1^4 + a_2x_1^3x_2 + a_3x_1^2x_2^2 + a_4x_1x_2^3 + a_5x_2^4$$

Calculate the derivation of the system along its paths.

$$\begin{aligned} \dot{V}(\mathbf{x}) = {} & (-4a_1 + 3a_2)x_1^6 + (-3a_2 + 6a_3)x_1^5x_2 \\ & + (-2a_3 + 9a_2)x_1^4x_2^2 + (-8a_1 - 3a_2 - a_4 \\ & + 12a_5)x_1^3x_2^3 + (-6a_2 - 6a_3)x_1^2x_2^4 \\ & + (-4a_3 - 9a_4)x_1x_2^5 + (-2a_4 - 12a_5)x_2^6 \end{aligned}$$

By replacing the above phrase with a negative phrase such as -12x16 - 24x26, we can obtain the following unique solutions for the coefficients:

a1 = 3, a2 = a3 = a4 = 0, a5 = 2

Thus, we obtain the following Lyapunov function.

$$V(x) = 3x_1^4 + 2x_2^4$$

This clearly shows the equilibrium point's asymptotic stability. As demonstrated in this example, the number of required equations was also adequate to determine the unique factors of the unknown coefficients. And so the Lyapunov function was discovered. However, there is no guarantee that the equations produced from the unification of a derivative function with a desired negative definite phrase provide a unique result. In the following section, we will explain the preceding example in a more broad context, resulting in a method for determining the Lyapunov function. The method described here is easily adaptable to more general systems.

Think about the following: degree 3 nonlinear homogeneous system.

$$\begin{cases} \dot{x}_1 = b_{11}x_1^3 + b_{12}x_2^3 \\ \dot{x}_2 = b_{21}x_1^3 + b_{22}x_2^3 \end{cases}$$

It called Equation (15) in this research

To begin, select a homogeneous Lyapunov candidate function of degree 4 as follows:

$$\begin{aligned} V(\mathbf{x}) &= a_1x_1^4 + a_2x_1^3x_2 + a_3x_1^2x_2^2 + a_4x_1x_2^3 + a_5x_2^4 \\ &= \mathbf{a}^T(\mathbf{x}^{[4]})^T \end{aligned}$$

It called Equation (16) in this research

Deriving this function in time gives:

$$\begin{aligned} \dot{V}(\mathbf{x}) = {} & 4\dot{x}_1x_1^3a_1 + (3\dot{x}_1x_1^2x_2 + x_1^3\dot{x}_2)a_2 \\ & + (2\dot{x}_1x_1x_2^2 + 2x_1^2\dot{x}_2x_2)a_3 \\ & + (\dot{x}_1x_2^3 + 3x_1\dot{x}_2x_2^2)a_4 + 4\dot{x}_2x_2^3a_5 \end{aligned}$$

It called Equation (17) in this research

When the dynamics of Eq. (15) are included into V, the preceding statement is reduced to:

$$\begin{aligned} \dot{V}(\mathbf{x}) = {} & (4b_{11}a_1 + b_{21}a_2)x_1^6 + (3b_{11}a_2 + 2b_{21}a_3)x_1^5x_2 \\ & + (2b_{11}a_3 + 3b_{21}a_2)x_1^4x_2^2 + (4b_{12}a_1 + b_{22}a_2 + b_{11}a_4 \\ & + 4b_{21}a_5)x_1^3x_2^3 + (3b_{12}a_2 + 2b_{22}a_3)x_1^2x_2^4 \\ & + (2b_{12}a_3 + 3b_{22}a_4)x_1x_2^5 + (b_{12}a_4 + 4b_{22}a_5)x_2^6 \end{aligned}$$

It called Equation (18) in this research

Which is obtained in the matrix format:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}^T \begin{bmatrix} 4b_{11} & 0 & 0 & 4b_{12} & 0 & 0 & 0 \\ b_{21} & 3b_{11} & 0 & b_{22} & 3b_{12} & 0 & 0 \\ 0 & 2b_{21} & 2b_{11} & 0 & 2b_{22} & 2b_{12} & 0 \\ 0 & 0 & 3b_{21} & b_{11} & 0 & 2b_{22} & b_{12} \\ 0 & 0 & 0 & 4b_{21} & 0 & 0 & 4b_{22} \end{bmatrix} \begin{bmatrix} x_1^6 \\ x_1^5x_2 \\ x_1^4x_2^2 \\ x_1^3x_2^3 \\ x_1^2x_2^4 \\ x_1x_2^5 \\ x_2^6 \end{bmatrix} = \mathbf{a}^T H^T (\mathbf{x}^{[6]})^T$$

It called Equation (19) in this research

$$H = \begin{bmatrix} 4b_{11} & b_{21} & 0 & 0 & 0 \\ 0 & 3b_{11} & 2b_{21} & 0 & 0 \\ 0 & 0 & 2b_{11} & 3b_{21} & 0 \\ 4b_{12} & b_{22} & 0 & b_{11} & 4b_{21} \\ 0 & 3b_{12} & 2b_{22} & 0 & 0 \\ 0 & 0 & 2b_{12} & 2b_{22} & 0 \\ 0 & 0 & 0 & b_{12} & 4b_{22} \end{bmatrix}$$

It called Equation (20) in this research

We now equalize Eq. (19) with a negative function, such as ZT(x[p+k-1])T :

$$\dot{V}(\mathbf{x}) \equiv Z^T(\mathbf{x}^{[p+k-1]})^T$$

It called Equation (21) in this research

In the two-variable mode n = 2, for example, the decision for Z can be as follows:

$$Z = \begin{bmatrix} -1 & 0 & -1 & 0 & \cdots & 0 & -1 \end{bmatrix}^T$$

It called Equation (22) in this research

As a result of comparing Equation. (19) and (21), the unknown coefficients a can be derived by solving the following equation.

Ha = Z

It called Equation (23) in this research

According to the coefficients of the basic functions, since the number of unknowns (ai) (the vector length of x[p]) in the resulting Equation (23) is less than the number of equations (that is, the vector length of x[p+k1]) and the coefficients are not exactly determined, one method to determine the optimal of these abnormalities is to use The least squares method, which proposes the following response from Equation 23.

$a = (H^T H)^{-1} H^T Z$

It called Equation (24) in this research.

Following the calculation of the a′s coefficients, the sign of V (x) should be confirmed. In the event that the required signs are not obtained, two options are suggested:

1. Reselect the Z vector, then confirm the required sign for theV (x) and assign theV(x) sign.
2. Increase the degree of Lyapunov's function and then repeat the preceding algorithm stages.

Based on Theorem 1, another approach for estimating the unknown coefficients of the Lyapunov candidate function is described below. Consider V(x)to the form of Equation (7), where we set the value of the functionV (x) on the unit sphere to a specified value. We consider the value of V (x) = 1 based on the homogeneity of this function. We can now write the following equations by selecting the number of N points on the surface of the unit sphere:

$$\sum_{i=1}^{m} a_i \dot{v}_i(x^j) = -1, \quad j = 1, 2, 3, \ldots, N$$

It called Equation (25) in this research

Where xj is the j-point on a unit sphere, and we have N equations and m unknown parameters, the set of Equations must always be N m solvable. It is worth noting that these N points are chosen in the shape of an equal align on the sphere, so that as their number increases, the points' locations become closer together, and therefore the precision of the solution improves. Increasing N, on the other hand, will result in heavier computing. That it will cause the program to lag. The set of Eq. (25) can be found in the matrix below:

$$\mathbf{Ga} = \begin{bmatrix} \dot{v}_1(\mathbf{x}^1) & \dot{v}_2(\mathbf{x}^1) & \cdots & \dot{v}_m(\mathbf{x}^1) \\ \dot{v}_1(\mathbf{x}^2) & \dot{v}_2(\mathbf{x}^2) & \cdots & \dot{v}_m(\mathbf{x}^2) \\ \vdots & \vdots & \ddots & \vdots \\ \dot{v}_1(\mathbf{x}^N) & \dot{v}_2(\mathbf{x}^N) & \cdots & \dot{v}_m(\mathbf{x}^N) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}$$

It called Equation (26) in this research

G matrix is a N mmatrix. Given that the number of equations in this device exceeds the number of unknowns, one technique of determining the missing vector is to use the Least Squares method. Simply put, the answer is as follows:

$$\mathbf{a} = (G^T G)^{-1} G^T \begin{pmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{pmatrix}$$

It called Equation (27) in this research

We must confirm the sign V (x) and V(x) after computing x, vector of the coefficients, by inserting them in theV (x) expression again. After ensuring that V (x) is negative, we can assess the stability of the analyzed system's equilibrium point by utilizing the estimated coefficients an of Eq. (27) and their replacement in Eq. (7), function V(x) forms that by determining its sign and applying Theorem 1.

4. CONCLUSION

A standard method for analyzing the stability of nonlinear systems is the Lyapunov stability and instability theorems. This paper presents these cases in the form of a theorem based on determining the sign of the initial function by considering the common part of the previous theorems, This is the inverse of the Lyapunov candidate function's derivative. Therefore, the approach of this paper, instead of focusing on the positive definiteness of the meaning of the Lyapunov candidate function, emphasizes on the negative definiteness of its derivative in the analysis of the stability of nonlinear systems. In this method, the Lyapunov candidate function is formed as a linear combination of a number of basic functions - a p-form with unknown coefficients. It is necessary to find coefficients of this linear composition so that derivative of candidate function Lyapov is negative definite function. Two methods have been introduced for calculating the unknown coefficients in homogeneous systems. While we have made strides in finding Lyapunov function in a nonlinear case, the extension to other nonlinear cases is possible. The development of stability analysis for a new nonlinear system should be included in further work. In addition, we are able to implement this solution in any branch of science.

5. ACKNOWLEDGMENT

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6. RECOMMENDATION

For further research, we recommend that the data collection and data analysis process be carried out more thoroughly in order to obtain maximum results.

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