



Error patterns in solving ordinary differential equations: A SOLO taxonomy-based analysis of pre-service mathematics teachers

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Abstrak

Penelitian ini bertujuan untuk menganalisis kesalahan mahasiswa dalam menyelesaikan soal Persamaan Diferensial Biasa (PDB) berdasarkan taksonomi SOLO (*Structure of the Observed Learning Outcome*). Penelitian ini adalah penelitian deskriptif kualitatif dengan subjek penelitian sebanyak 29 mahasiswa Program Studi Pendidikan Matematika Universitas Mataram. Data dikumpulkan melalui tes uraian dan wawancara, kemudian dianalisis menggunakan klasifikasi taksonomi SOLO dan jenis kesalahan menurut Subanji dan Mulyoto. Hasil penelitian menunjukkan bahwa sebagian besar mahasiswa berada pada level kognitif pre-structural, uni-structural, dan multi-structural dengan dominasi kesalahan konseptual dan teknis. Kesalahan umum meliputi penggunaan rumus yang tidak tepat, kesalahan manipulasi aljabar, dan lemahnya keterampilan kalkulus dasar. Minimnya pemahaman terhadap konsep prasyarat seperti aljabar dan kalkulus menjadi faktor utama penyebab kesalahan. Temuan ini menunjukkan pentingnya penguatan konsep dasar dan strategi pembelajaran berbasis taksonomi SOLO untuk meningkatkan pemahaman mahasiswa terhadap PDB.

Kata Kunci: persamaan diferensial biasa; taksonomi SOLO; kesalahan konseptual, kesalahan teknis

Abstract

This study aims to analyze students' errors in solving Ordinary Differential Equations (ODE) problems based on the SOLO taxonomy (*Structure of the Observed Learning Outcome*). This qualitative descriptive research involved 29 students from the Mathematics Education Study Program at the University of Mataram. Data were collected through essay tests and interviews, then analyzed using the SOLO taxonomy classification and error types based on the framework by Subanji and Mulyoto. The results show that most students were at the pre-structural, uni-structural, and multi-structural cognitive levels, with a dominance of conceptual and technical errors. Common mistakes included the use of incorrect formulas, errors in algebraic manipulation, and weak fundamental calculus skills. A lack of understanding of prerequisite concepts such as algebra and calculus was identified as the main factor contributing to these errors. These findings highlight the importance of strengthening basic concepts and implementing learning strategies based on the SOLO taxonomy to improve students' understanding of ODE topics.

Keywords: ordinary differential equations; SOLO taxonomy; conceptual errors; technical errors

1. INTRODUCTION

The Differential Equations (DE) course is a crucial component in the undergraduate Mathematics Education curriculum. Generally, the course is divided into two main parts: Ordinary Differential Equations (ODE) and Partial Differential Equations (PDE). Key topics in ODE include linear and nonlinear DEs, homogeneous and non-homogeneous DEs, first-order linear DEs, and second- or higher-order DEs.

ODEs play a significant role in solving various contextual problems in everyday life, such as modeling bacterial growth, cooling and heating processes, and other dynamic phenomena (Luneta & Makonye, 2010). herefore, the ability to accurately solve ODEs is essential, particularly in supporting mathematically-informed decision-making across multiple fields of application. However, previous studies have shown that ODEs remain a conceptual and procedural challenge for many university students (Ajudin et al., 2021; Luneta & Makonye, 2010; Maat & Zakaria, 2011). Farlina et al. (2018) also confirmed that students still struggle to solve ODE problems comprehensively.

According to Nykamp (2015), solving ODEs is often more complex than basic integration. The most challenging part for students typically lies in selecting the appropriate integration technique based on the structure of the equation (Makamure & Jojo, 2022). The wide variety of ODE types—such as separable, exact, linear, homogeneous, and non-homogeneous forms, as well as integrating factors—requires a deep understanding of the underlying concepts. However, preliminary studies and observations reveal that students' mastery of these materials remains suboptimal. This is evident from the frequent errors found in solving ODE problems, especially those involving higher-order differential equations.

To pinpoint students' weaknesses more precisely, it is necessary to analyze the types and forms of errors they make. This process not only helps portray their level of content mastery but also serves as a foundation for designing more effective instructional interventions. One relevant approach to describing the quality of student responses is the SOLO (Structure of the Observed Learning Outcome) taxonomy, which classifies learning outcomes into five levels: prestructural, unistructural, multistructural, relational, and extended abstract. In this study, the SOLO taxonomy is used to identify students' understanding of ODE topics and as a framework for analyzing their errors. The classification of errors follows the typology proposed by Subanji and Mulyoto (Yarman et al., 2020) which categorizes the types of errors made by students in solving ODE problems.

In addition to the conceptual challenges inherent in ODE material, students often face difficulties in understanding and applying foundational calculus concepts, especially in using the chain rule. These difficulties directly impact their performance in solving ODEs, as the topic heavily relies on calculus as its foundation (Makonye, 2016). Therefore, a

thorough analysis of student errors is crucial to identifying the root causes of learning difficulties.

Table 1. Types of Errors in Solving ODE Problems

Error Type	Indicators
Concept Error (CE)	(i) Incorrectly determining the theorem or formula to use; (ii) Using a theorem or formula inappropriately or without stating it properly.
Error Using Data (ED)	(i) Failing to use relevant data; (ii) Inaccurately assigning data to variables; (iii) Adding unnecessary data.
Language Interpretation Error (LE)	(i) Incorrectly translating everyday language into mathematical language; (ii) Misinterpreting symbols, graphs, or tables.
Technical Error (TE)	(i) Computational errors; (ii) Errors in manipulating algebraic operations.
Error making conclusions (EC)	i) Drawing conclusions without appropriate supporting ustification; (ii) Making invalid logical inferences.

This study highlights the importance of exploring students' errors and misconceptions when working on ODE problems. By mapping the most frequent types of errors, more targeted learning strategies can be developed to meet student needs. Identifying these error types also supports the design of more effective and adaptive instructional methods based on students' levels of content mastery. Thus, this study aims to investigate the conceptual and procedural difficulties and errors exhibited by students in solving ODE problems. The findings are expected to contribute to improving the quality of instruction and enhancing students' understanding of ODE material more comprehensively.

2. RESEARCH METHOD

This study is a qualitative descriptive research that aims to describe the types of errors made by students in solving Ordinary Differential Equations (ODE) problems, based on the classification of the SOLO taxonomy (Structure of the Observed Learning Outcome). The research was conducted in the Mathematics Education Study Program at the University of Mataram.

The subjects of this study were 29 students who were enrolled in the Differential Equations course and exhibited errors in solving the given problems. Data collection was carried out in two stages: a written test and interviews. The test instrument consisted of three open-ended questions designed to explore students' abilities in solving ODE problems. The test results were analyzed to identify the types and levels of errors made by the students. Subsequently, interviews were conducted with selected participants to confirm the identified errors and to explore the underlying causes.

The data analysis in this study followed three steps: (1) data reduction, (2) data presentation, and (3) conclusion drawing or verification. The SOLO taxonomy was used as the framework for classifying students' levels of thinking, which consists of five

categories: prestructural, unistructural, multistructural, relational, and extended abstract. The classification criteria for each SOLO level are presented in Table 1 below..

Table 1. Criteria for Students' Thinking Levels Based on SOLO Taxonomy

SOLO Level	Criteria
Level 1: Pre Structural	- Students show no understanding and use irrelevant information and/or fail to grasp the topic.. - Knowledge is fragmented, disorganized, and lacks meaningful connection to the topic.
Level 2: Uni-Structural	- Students handle one relevant aspect and make a simple connection. - They can use terminology, recall facts, follow basic instructions, paraphrase, identify, or compute.
Level 3: Multi-Structural	- Students address multiple aspects, but treat them as separate and unrelated. - They can calculate, describe, classify, apply methods, organize, or execute procedures.
Level 4: Relational Level	- Students understand the relationships between several aspects and how they form a coherent whole. - They can compare, connect, analyze, apply theory, and explain causal relationships.
Level 5: Extended Abstract	- Students can generalize beyond the given content, view structures from multiple perspectives, and transfer ideas to new contexts. - They can hypothesize, theorize, critique, or generalize.

Source: (Putri et al., 2017)

3. RESULTS AND DISCUSSION

3.1 Results

This study reveals variations in students' error rates when solving Ordinary Differential Equation (ODE) problems, classified according to the SOLO taxonomy. The percentage of students committing errors at each SOLO level for each question is presented in Table 3.

Table 3. Percentage of student errors at each SOLO level

Question	Pre-structural	Uni-structural	Multi-structural	Relational	Extended abstracts
Q1	100.00%	100.00%	50.00%	42.86%	-
Q2	87.50%	100.00%	100.00%	5.56%	-
Q3	100.00%	80.00%	9.52%	0.00%	100.00%

Note: The numbers represent the percentage of participants who made specific errors; a dash (–) indicates that the question did not assess that particular level of the SOLO taxonomy.

The data reveal that almost all participants made errors at the pre-structural and uni-structural levels, especially on Q1 and Q3. By contrast, few students reached the relational or extended-abstract levels, indicating a limited ability to integrate and apply concepts holistically in solving ODEs.

Error types were analyzed following Subanji and Mulyoto's classification: conceptual errors, data usage errors, language interpretation errors, technical errors, and conclusion-drawing errors. Table 4 summarizes the percentage of each error type per question.

Table 4. Error types by question

Question	Conceptual Error	Error using data	Language interpretation error	Technical errors	Error making conclusions
Q1	55.17%	41.38%	6.90%	62.07%	48.28%
Q2	17.24%	13.79%	17.24%	20.69%	3.45%
Q3	82.76%	10.34%	6.90%	82.76%	44.83%

Analysis shows technical errors were the most prevalent, particularly in Q1 and Q3, indicating poor algebraic manipulation skills. These weaknesses directly affected students' abilities to perform variable separation and determine characteristic equations in ODE solving.

In summary, limited mastery of fundamental calculus and algebra concepts presented a major barrier to accurate ODE problem-solving. These findings underscore the need for targeted instructional strategies focusing on strengthening conceptual knowledge and technical skills in ODE contexts.

Description of Pre-Service Teachers' Errors at the Pre-structural Level

Students at this lowest level displayed minimal understanding and made various errors: conceptual error (CE), error using data (ED), language interpretation error (LE), technical error (TE), and error making conclusions (EC). Table 5 provides error frequencies.

Pada level *pre-structural*, mahasiswa menunjukkan ketidaktahuan atau pemahaman yang sangat terbatas terhadap konsep-konsep dasar dalam penyelesaian soal Persamaan Diferensial (PD). Jenis kesalahan yang dominan pada level ini mencakup *conceptual error* (CE), *error using data* (ED), *language interpretation error* (LE), *technical error* (TE), dan *error making conclusions* (EC). Rincian jumlah kesalahan berdasarkan jenis dan pertanyaan dapat dilihat pada Tabel 5.

Table 5. Frequency of Error Types Made by Pre-Service Teachers at the Pre-Structural Level

Question	Error Types				
	CE	ED	LE	TE	EC
Q1	3	3	2	4	3
Q2	1	1	0	1	0
Q3	1	1	0	1	0

Figure 1 below shows an example of a pre-service teacher's response classified at the pre-structural level in solving Question 1.

PD linear orde 1
 $2x dx + y dy = 0$

1. Nyatakan pd orde 1 dalam bentuk standar: $y' + p(x)y = Q(x)$.

$y dy = -2x dx$

2. Hitung faktor integral.
 $u(x) = e^{\int p(x) dx}$
 $u(x) = e^{\int -2x dx}$

Errors occurred in expressing the equation in standard form suitable for applying the integrating factor method

3. Masukkan kebaruan pada langkah 1 dengan $u(x)$.
 $u(x) [y' + p(x)y] = u(x) \cdot Q(x)$
 $u(x) \cdot [y dy] = u(x) \cdot -2x dx$

4. Kita bisa pakai persamaan di langkah 3 karena dengan $\frac{d}{dx} [u(x)y] = y(x) \cdot Q(x)$

5. Integralkan kedua ruas diperoleh solusi umum.
 $y = \frac{1}{u(x)} \left[\int u(x) Q(x) dx + c \right]$
 $y = \frac{1}{u(x)} \left[\int \frac{1}{2} (-2x^2 + c)^{-\frac{1}{2}} \right]$
 $y = 2 \sqrt{-2x^2 + c}$

There were errors in the solution process

Figure 1. Example of an Error Made by Subject S2 at the Pre-Structural Level for Question 1

An analysis of Subject S2's response reveals several interrelated types of errors: (a) Error in Data Usage (ED): The subject failed to appropriately utilize relevant information from the question. They attempted to solve the problem using the method of integral factorization, which was not suitable for the given problem type; (b) Conceptual Error (CE): This error involves two aspects: (1) incorrect selection of a solution method, in which the subject used an approach that did not align with the general form of the differential equation; and (2) the application of formulas or theorems without fulfilling the necessary conditions for their use. It appears that the subject insisted on using a familiar method due to limited knowledge of alternative and more appropriate strategies; (c) Technical Error (TE): This includes mistakes in numerical calculations and algebraic manipulations. Such errors indicate weak procedural skills in handling the mathematical expressions underlying the solution process of differential equations and; (d) Error in Conclusion (EC): The subject wrote the final answer without presenting a coherent step-by-step solution. This was likely due to an awareness that the applied method was incorrect, combined with a lack of understanding of the correct steps to take.

Overall, the response of the subject at the pre-structural level indicates that the pre-service teacher had not yet developed adequate conceptual or procedural understanding in solving differential equation problems, even at the initial stages of the solution. This finding highlights the need to strengthen basic conceptual understanding, the ability to

select appropriate problem-solving strategies, and practice in algebraic manipulation as a foundation for students' technical skills.

Description of Pre-Service Teachers' Errors at the Uni-Structural Level

At the uni-structural level, pre-service teachers generally begin to demonstrate an understanding of a single aspect or part of the problem, but are not yet able to connect that aspect with other elements in a comprehensive manner. The errors that appear at this level primarily include conceptual errors (CE), errors in using data (ED), and technical errors (TE). The detailed frequency of these errors by type and question is presented in Table 6.

Table 6. Frequency of Error Types Made by Pre-Service Teachers at the Uni-Structural Level

Question	Error Types				
	CE	ED	LE	TE	EC
Q1	6	8	0	8	7
Q2	1	1	0	1	0
Q3	2	1	0	4	3

Figure 2 shows an example of an error made by a pre-service teacher with the initials S14 in solving Question 3, which was classified at the uni-structural level.

solusi dari PD non homogen
 $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$
 jawab :
 PD homogen : $y'' - 6y' + 9y = 0$
 Pers. bantu : $(r^2 + 6r + 9) = 0$
 $(r+3)(r+3) = 0$
 $r = -3 \vee r = -3$
 Solusi umum PD homogen :
 $y_h = C_1 e^{-3x} + C_2 e^{-3x}$
 Solusi khusus tak homogen
 $y_p = Ax^2 + B - C e^{3x}$
 $y_p = 6x^2 + 2 - 12e^{3x}$
 $y'_p = 12x - 12e^{3x} \cdot 12x$
 $y''_p = 12 - 12e^{3x} \cdot 12$

Error in formulating the auxiliary equation

Figure 2. Example of an Error Made by Subject S14 at the Uni-Structural Level for Question 3

Based on the results of the analysis and interviews, Subject S14 exhibited several types of errors as follows: (a) Technical Errors (TE): (1) Calculation error, the subject incorrectly determined the roots of the characteristic equation, which affected the final result, even though the subsequent steps were carried out correctly. The interview confirmed that this error was due to a lack of careful reading of the question, caused by working in a rush. (2) Algebraic manipulation error, the subject made mistakes in operating algebraic

expressions used during the solution process, indicating weak basic symbolic manipulation skills; (b) Conceptual Errors (CE): (1) Incorrect application of formulas or theorems, the subject applied formulas without considering the necessary conditions required for their proper use and (2) Error in selecting a solution strategy, the steps taken did not align with the appropriate approach for the given type of differential equation.

These findings indicate that at the uni-structural level, pre-service teachers are beginning to recognize solution procedures, but their limited understanding prevents them from accurately integrating concepts. This limitation is evident in their tendency to memorize procedures without fully understanding the meaning and conditions for their application, as well as their weak foundational algebra skills, which contribute to both technical and conceptual errors.

Description of Pre-Service Teachers' Errors at the Multi-Structural Level

Pre-service teachers at the multi-structural level are generally able to identify and apply several key components in solving problems, but they are not yet able to integrate this information holistically. At this level, the most common types of errors include conceptual errors (CE), errors in using data (ED), technical errors (TE), and errors in making conclusions (EC). The detailed number of errors for each type and question is presented in Table 7.

Table 7. Frequency of Error Types Made by Pre-Service Teachers at the Multi-Structural Level

Question	Error Types				
	CE	ED	CE	TE	CE
Q1	5	1	0	4	3
Q2	5	1	0	4	3
Q3	22	1	0	22	9

The analysis results show that conceptual errors (CE) and technical errors (TE) are the most dominant types of errors at this level, particularly in Question 3. This indicates that although pre-service teachers are able to recognize the basic structure of the problem, they still struggle to accurately apply concepts and perform correct algebraic manipulations.

As an illustration, Figure 3 presents an example of an error made by Subject S5 in solving Question 2, which involves a first-order linear differential equation.

$xy' + y = e^x, x > 0$
 Jawab:
 • ubah ke bentuk umum
 $x \frac{dy}{dx} + y = e^x$
 • diperoleh $P(x) = 1$
 $Q(x) = e^x$
 • Faktor Integral: $e^{\int 1 dx} = e^x$
 • kalikan kedua ruas dengan faktor Integral
 $e^x \cdot x \cdot \frac{dy}{dx} + y \cdot e^x = e^x \cdot e^x$

Error in constructing the standard form of the linear equation

$x e^x \frac{dy}{dx} + y e^x = e^{2x}$
 • Nyatakan ruas kiri sbg bentuk umum:
 $\frac{d}{dx} (y e^x) = e^{2x}$
 • Integrasikan kedua ruas:
 $y e^x = \int e^{2x} dx$
 $y e^x = \frac{1}{2} e^{2x} + C$
 $y e^x = \frac{1}{2} e^{2x} + C$
 $y = \frac{1}{2} e^{2x} + C$
 $y = \frac{1}{2} e^{-x} + C e^{-2x}$

Figure 3. Example of an Error Made by Subject S5 at the Multi-Structural Level for Question 2

Subject S5 exhibited two main types of errors as follows: (a) Conceptual Error (CE): The subject applied the integrating factor formula $e^{\int P(x)dx}$ to solve the differential equation without first verifying that the equation was in the standard form $y' + P(x)y = Q(x)$. In this case, the given differential equation was $x \frac{dy}{dx} + y = e^x$, which had not yet been expressed in the required standard form. The subject directly applied the formula without transforming the equation, resulting in an incorrect integrating factor that affected the subsequent solution steps; (b) Technical Error (TE): In addition to misapplying the formula, the subject also demonstrated weaknesses in algebraic manipulation, such as errors in setting up the integral or simplifying mathematical expressions, which contributed to an incorrect final result.

These findings indicate that although pre-service teachers at the multi-structural level have mastered several steps in the problem-solving process, a lack of understanding of the formal structure of differential equations and weaknesses in technical aspects lead to inaccuracies in their solutions. This highlights the importance of simultaneously strengthening both conceptual and procedural aspects in the teaching of ordinary differential equations (ODEs).

Description of Pre-Service Teachers' Errors at the Relational Level

Pre-service teachers at the relational level are generally able to connect multiple concepts and procedures in problem-solving and demonstrate a more integrated understanding. However, errors are still observed at this level, particularly technical errors and errors in drawing conclusions. The types of errors identified at this level include technical errors (TE) and errors in making conclusions (EC). The detailed number of errors by question is presented in Table 8.

Table 8. Frequency of Error Types Made by Pre-Service Teachers at the Relational Level

Question	Error Types				
	CE	ED	CE	TE	CE
Q1	0	0	0	3	0
Q2	0	0	0	1	1
Q3	0	0	0	0	0

The analysis indicates that although pre-service teachers were able to identify and connect relevant concepts in problem-solving, they still encountered difficulties in technical aspects, such as calculations and algebraic manipulations, as well as in formulating conclusions supported by proper mathematical reasoning. As an illustration, Figure 4 presents an example of an error made by Subject S25 in Question 2.

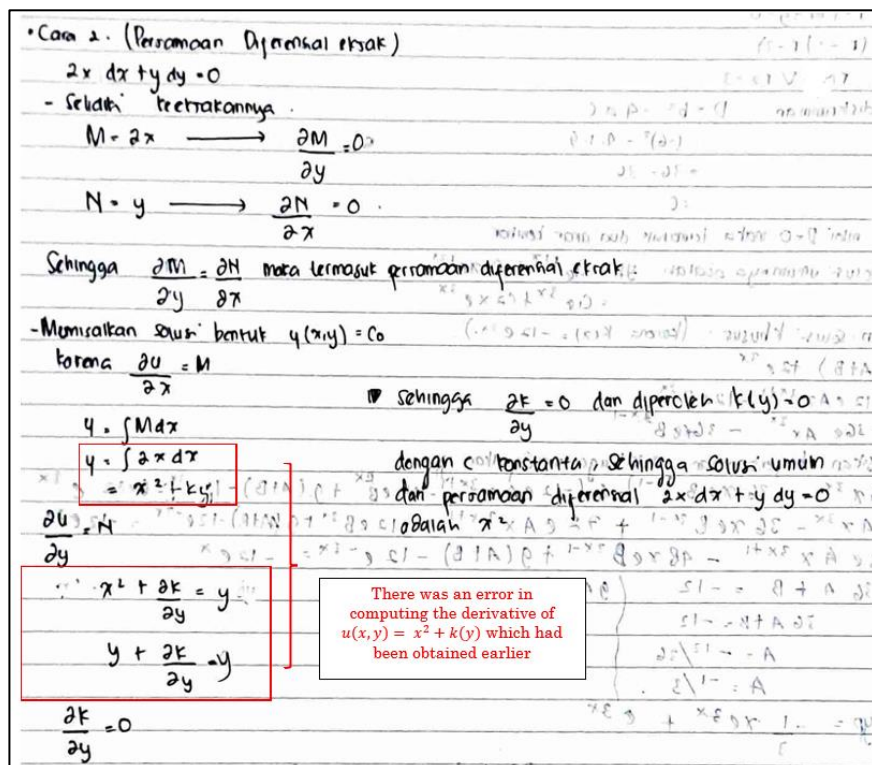


Figure 4. Example of an Error Made by Subject S25 at the Relational Level for Question 2

Based on the analysis of the response and interview results, Subject S25 demonstrated two types of errors: (a) Technical Error (TE): This includes inaccuracies in calculations and errors in manipulating algebraic expressions. Although the subject conceptually understood the solution steps, miscalculations led to an incorrect final answer. This suggests that weaknesses in basic procedural skills remain an obstacle, even when conceptual understanding is already established; (b) Error in Making Conclusions (EC): The subject drew a final conclusion without providing clear justification or mathematical

reasoning. This indicates that the student is not yet fully accustomed to constructing logical arguments to support their solution..

Overall, errors at the relational level tend to be more mechanical in nature and relate to precision as well as the ability to present arguments systematically. These findings highlight the importance of practice that emphasizes procedural accuracy and mathematical communication skills in problem solving, particularly in the topic of Ordinary Differential Equations.

Description of Pre-Service Teachers' Errors at the Extended Abstract Level

Pre-service teachers at the extended abstract level demonstrate higher-order thinking skills, where they are not only able to understand and integrate various concepts in problem-solving but can also generalize and develop new approaches within broader contexts. However, despite operating at a complex cognitive level, some errors are still observed—particularly in conceptual and technical aspects. The detailed number of errors by type and question is presented in Table 9.

Table 9. Frequency of Error Types Made by Pre-Service Teachers at the Extended Abstract Level

Question	Error Types				
	CE	ED	CE	TE	CE
Q1	0	0	0	0	0
Q2	0	0	0	0	0
Q3	2	0	0	2	0

As shown in Table 9, errors at this level were found only in Question 3, with a predominance of conceptual errors (CE) and technical errors (TE). This indicates that although pre-service teachers have demonstrated deep understanding and reflective thinking skills, they remain vulnerable to errors caused by a lack of precision and time management when solving complex problems. Figure 5 presents an example of an error made by Subject S23 in answering Question 3.

Based on the analysis, Subject S23 exhibited two types of errors: (a) Conceptual Error (CE): The subject used an inappropriate formula in solving the problem, indicating a misconception about the conditions for applying the formula. The subject also made an error in selecting the most relevant solution strategy for the given problem; (b) Technical Error (TE): A calculation error occurred that affected the accuracy of the final result, even though the overall solution process followed a generally correct sequence.

$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$
 → Pers. bantu PD homogen
 $r^2 - 6r + 9 = 0$
 $a=1, b=-6, c=9$
 $r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$
 $= \frac{6 \pm \sqrt{36 - 36}}{2}$
 $r_{1,2} = 3$ (real kembar)
 $y_h = c_1 e^{3x} + c_2 x e^{3x}$
 → $y_p = (Ax^2 + B) - 12e^{3x}$
 $y_p' = 2Ax - 12e^{3x}$
 $y_p'' = 2A - 12e^{3x}$
 → Substitusi ke PD
 $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$
 $(2A - 12e^{3x}) - 6(2Ax - 12e^{3x}) + 9(Ax^2 + B) - 12e^{3x} = 6x^2 + 2 - 12e^{3x}$
 $2A - 12e^{3x} - 12Ax + 72e^{3x} + 9Ax^2 + 9B - 12e^{3x} = 6x^2 + 2 - 12e^{3x}$
 $9A = 6 \Leftrightarrow A = \frac{6}{9} = \frac{2}{3}$
 $9B = 2 \Leftrightarrow B = \frac{2}{9}$
 $y_p = \left(\frac{6}{9}x^2 + \frac{2}{9} \right) - 12e^{3x}$
 $y = y_p + y_h \Leftrightarrow y = \frac{6}{9}x^2 + \frac{2}{9} - 12e^{3x} + c_1 e^{3x} + c_2 x e^{3x}$

An incorrect determination of the particular solution for the non-homogeneous differential equation $k(x) = 6x^2 + 2 - 12e^{3x}$ resulted in further errors in the next steps

Figure 5. Example of an Error Made by Subject S23 at the Extended Abstract Level for Question 3

Interview results indicated that these errors were primarily caused by non-cognitive factors, such as poor time management during problem-solving and a lack of thoroughness in reviewing the solution steps.

Overall, the errors at the extended abstract level were not fundamental in nature but rather mechanical and situational. These findings emphasize that even among students with high-level thinking skills, accuracy and efficiency remain crucial factors in producing correct and complete solutions.

3.2 DISCUSSION

The results of this study generally indicate that pre-service teachers experience various difficulties in learning Ordinary Differential Equations (ODEs), which stem from multiple factors. Some of the errors identified were procedural or technical in nature, arising from mistakes in applying concepts that were actually already known. However, the majority of the errors were classified as conceptual or comprehension errors, indicating a lack of knowledge within specific contexts (Brown et al., 2014).

The analysis based on the SOLO taxonomy reveals that most students operate at low levels of cognitive skills—namely, the pre-structural, uni-structural, and multi-structural levels—as described by Tarigan et al. (2019). Only a few students reached the relational level, and very few demonstrated thinking abilities at the extended abstract level. This

reflects the generally low performance of students in solving ODE problems. These findings are consistent with the study by Maat & Zakaria (2011) which reported that many learners struggle with ODEs and related topics.

The errors found in this study can be grouped into two main categories: individual errors and errors commonly found across different levels of student ability. Based on the test and interview results, it was identified that many students had not yet mastered the necessary mathematical skills to solve ODEs effectively (Yarman et al., 2020).

More specifically, conceptual errors emerged as the most dominant type. This indicates that students lack a strong understanding of the prerequisite concepts underlying the solution of ODEs, such as basic algebra and calculus skills. Legutko (2008) argues that conceptual errors reflect a limited knowledge base, which is closely associated with a lack of imagination and creativity in dealing with new situations. The implication of these findings is that students do not yet possess sufficient mastery of the factual knowledge, concepts, and basic skills required to solve ODE problems successfully.

One of the main difficulties faced by students is identifying the type of differential equation and determining the appropriate solution method. For instance, when dealing with a first-order linear differential equation in the form $\frac{dy}{dx} + P(x)y = Q(x)$, students often make mistakes in determining the integrating factor, which should be found using the formula $e^{\int P(x)dx}$. Failure to recognize this standard form leads to a cascade of errors throughout the solution process. Nykamp (2015) emphasizes that solving ODEs is far more complex than simply performing integration, as it requires a wide range of advanced mathematical skills.

Furthermore, Nykamp also highlights that one of the greatest challenges in solving ODEs lies in students' ability to choose the appropriate integration method—whether by substitution, integration by parts, or partial fractions. Since ODEs are an advanced topic that heavily depends on calculus mastery (Luneta & Makonye, 2010), weaknesses in prerequisite skills such as integration, differentiation, and algebraic manipulation are major contributors to poor student performance. Common errors include incorrect use of natural logarithmic rules, confusion between integrating constants and variables, and omissions of the constant of integration. These issues all point to a weak foundational understanding of calculus among students.

Herholdt & Sapire (2014), argue that the frequent recurrence of specific types of errors among students may indicate widespread conceptual confusion and the need for further clarification in instruction. Widespread algebraic errors also significantly impact students' ability to solve ODEs, especially during the processes of separating variables or constructing characteristic equations.

Furthermore, students' difficulties in solving ODE problems are also linked to their inability to connect advanced mathematical topics with the foundational material

previously taught. For example, many students are unable to relate skills such as simplifying algebraic expressions, factoring, and applying algebraic identities—typically learned in secondary school—to the context of solving ODEs at the university level. This fundamental weakness hinders their understanding of more complex concepts (Moradi et al., 2024).

In conclusion, students' errors in solving ODEs generally stem from two main sources: (1) Conceptual or comprehension errors—including poor algebra skills, inappropriate use of formulas, and misconceptions related to logarithmic rules and the structure of linear equations; and (2) Technical errors—such as carelessness and mistakes in calculations. These findings highlight the critical importance of equipping students with prerequisite skills, particularly in algebra and basic calculus, before introducing them to first-order differential equations.

4. CONCLUSION

This study revealed that most pre-service mathematics teachers operated at the lower levels of the SOLO taxonomy specifically at the pre-structural, uni-structural, and multi-structural stages. Only a few students demonstrated relational thinking, and very few reached the extended abstract level. The dominant error type identified was conceptual error, highlighting insufficient mastery of foundational concepts necessary for solving ordinary differential equations (ODEs), particularly those related to algebra and calculus. In addition, technical errors such as computational mistakes and flawed algebraic manipulations were frequently observed.

These findings contribute to a deeper understanding of the specific learning challenges faced by pre-service teachers when working with ODEs, especially in connecting prior mathematical knowledge to more advanced topics. The results emphasize the importance of integrating explicit concept reinforcement and error diagnosis into teacher education programs.

5. RECOMMENDATIONS

Future research is encouraged to develop instructional approaches that strengthen prerequisite conceptual understanding and implement SOLO taxonomy-based strategies to enhance students' thinking skills in solving ordinary differential equations (ODEs). Longitudinal studies and the integration of technology are also recommended to help reduce both technical and conceptual errors. Challenges such as limited sample size and variations in students' prior knowledge should be carefully considered in subsequent studies.

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