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A Simulation Study of Interval Estimation in Nonparametric Regression Using the Truncated Spline Estimator

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Abstract
This study examines interval estimation in truncated spline nonparametric regression using simulated data. The study aims to determine the impact of sample size, variance, and knot points on the performance of the truncated spline estimator. The results show that as the sample size increases, both the Generalized Maximum Likelihood (GML) and Mean Square Error (MSE) values decrease, while the coefficient of determination increases. This study also reveals that increasing the variance leads to higher GML and MSE values, as well as a lower coefficient of determination. Furthermore, the truncated spline nonparametric regression model achieves optimal performance with three knot points. The results showed that the more knot points, the GML and MSE values will decrease, while the coefficient of determination increases. The results of this study show that the determination of sample size, variance, and knot points significantly affects the accuracy and efficiency of the truncated spline nonparametric regression model, allowing it to serve as a reference for applying truncated spline nonparametric regression more effectively to produce a more optimal model that aligns with the characteristics of the data.

Keywords: GML; interval estimation; nonparametric regression; simulation study; spline truncated

Abstrak
Penelitian ini mengkaji terkait estimasi interval pada regresi nonparametrik spline truncated menggunakan data simulasi. Penelitian bertujuan untuk mengetahui dampak jumlah data, variansi, dan titik knot terhadap kinerja estimator spline truncated. Hasil penelitian menunjukkan dampak ketika jumlah data yang digunakan semakin besar, maka nilai *Generalized Maximum Likelihood* (GML) dan *Mean Square Error* (MSE) semakin menurun, sedangkan nilai koefisien determinasi semakin meningkat. Penelitian ini juga menunjukkan dampak ketika nilai variansi ditingkatkan, maka akan meningkatkan nilai GML dan MSE serta menurunkan nilai koefisien determinasi. Selain itu, diperoleh jumlah titik knot optimal pada model regresi nonparametrik spline truncated dengan tiga titik knot. Hasil penelitian menunjukkan bahwa semakin banyak titik knot, maka nilai GML dan MSE akan semakin menurun, sedangkan nilai koefisien determinasi semakin meningkat. Hasil dari penelitian ini memberikan gambaran bahwa penentuan jumlah data, variansi, dan titik knot sangat mempengaruhi akurasi dan efisiensi model regresi nonparametrik spline truncated, sehingga dapat menjadi acuan dalam penerapan regresi nonparametrik spline truncated secara lebih efektif agar menghasilkan model yang lebih optimal dan sesuai dengan karakteristik data.

Kata Kunci: estimasi interval; GML; regresi nonparametrik; studi simulasi; spline truncated

1. INTRODUCTION

Nonparametric regression is a regression analysis approach that helps to identify the relationship patterns between predictor and response variables without any prior information about the shape of the regression curve. (Sifriyani et al., 2023). The strength of nonparametric regression lies in exceptional flexibility, which allows the data to independently determine the shape of the regression curve on its own, without the influence of the researcher's biases (Ni'matuzzahroh & Dani, 2024). The nonparametric regression method has led to the creation of various estimators, including the truncated spline, kernel, Fourier series, and polynomial (Octavanny et al., 2021).

Currently, the truncated spline estimator is gaining popularity due to its flexibility. The truncated spline estimator is a polynomial function with a segmented nature over specific intervals (Adrianingsih et al., 2025). Knot points serve as junctions that indicate changes in data patterns over different intervals (Prawanti et al., 2019). Determining the optimal knot points is a crucial step in truncated spline nonparametric regression, too few knot points can make the model less capable of capturing complex data patterns. Conversely, too many knot points make the model overly complex, increasing the risk of overfitting (Dani et al., 2024). A method that can be utilized to find the optimal knot points is Generalized Maximum Likelihood (GML) (Wang, 1998). Several previous studies have implemented the truncated spline nonparametric regression approach, including research by (Nurcahayani et al. (2019), Dani et al. (2021), and Kuswanto et al. (2022). However, these studies are restricted to point estimation, which has limitations in delivering precise information about population parameters. To overcome this limitation, this study will expand point estimation by employing interval estimation.

Interval estimation expands upon point estimation by providing an estimated parameter value that is not limited to a single point but rather includes a range with lower and upper bounds (Suprapto, 2018). Interval estimation is considered stronger than point estimation because there is an interval in the parameter value used to estimate the population (Islamiyati et al., 2022). Previous studies that used interval estimation in truncated spline regression modeling include studies conducted by (Suprapto (2018), (Islamiyati et al. (2022), and (Setyawati et al. (2022). This study will focus on the interval estimation of truncated spline nonparametric regression using simulated data. Simulated data is preferred because it can produce a range of controlled conditions, allowing researchers to evaluate the estimation method's performance free from external influences. Previous studies that used simulated data in the truncated spline nonparametric regression approach include research conducted by Sudiarsa (2019), Dani et al. (2021), and Ratnasari et al. (2021).

Based on the provided explanation, this study focuses on interval estimation in truncated spline nonparametric regression using simulated data. Interval estimation provides additional information in the form of a confidence interval that is believed to contain the true value, thereby supporting more accurate decision-making. The study aims to determine the impact of the sample size, variance, and knot points used on the performance of the truncated spline estimator. The outcomes of this study are expected to act as a reference to enrich scientific insight and knowledge related to the use of truncated spline nonparametric regression with simulated data.

2. RESEARCH METHOD

2.1 Truncated Spline Nonparametric Regression

Nonparametric regression represents an approach in regression analysis that is utilized to detect patterns of relationships between predictor and response variables without any prior assumptions about the shape of the regression curve (Fitriyani et al., 2021). The most prevalent nonparametric regression estimator is the truncated spline, which can efficiently manage data characteristics that change across specific sub-intervals. Its advantages include strong visual and statistical interpretation, along with significant flexibility (Adrianingsih et al., 2021). The truncated spline function of order m and knot points $k_1, k_2, \ldots, k_r$ is written in Equation (1).

$$g(x_i) = \lambda_0 + \sum_{j=1}^{m} \lambda_j x_i^j + \sum_{q=1}^{r} \lambda_{m+q} \left(x_i - k_q \right)_+^m \quad (1)$$

The truncated function is given by Equation (2).

$$\left(x_i - k_q \right)_+^m = \begin{cases} \left(x_i - k_q \right)_+^m; & x_i \geq k_q \\ 0; & x_i < k_q \end{cases} \quad (2)$$

where λ is the estimated model parameter, k is the knot point denoted by r (Husain et al., 2021). Nonparametric regression that includes one response variable and multiple predictor variables is known as multivariable truncated spline nonparametric regression. The multivariable truncated spline nonparametric regression model is presented in Equation (3).

$$y_i = \lambda_0 + \sum_{s=1}^{t} \left(\sum_{j=1}^{m} \lambda_{sj} x_{si}^j + \sum_{q=1}^{r} \lambda_{s(m+q)} \left(x_{si} - k_{sq} \right)_+^m \right) + \varepsilon_i \quad (3)$$

where y_i is the response variable, x_{si}^j is the predictor variable, λ is the regression parameter coefficient, k is knot point, and ε is residual (Widyastuti et al., 2021).

2.2 Generalized Maximum Likelihood

The Generalized Maximum Likelihood (GML) method is applicable for selecting optimal knot points in truncated spline nonparametric regression and is particularly useful for handling correlated data (Wang, 1998). The formula for determining the number of knot points with the GML method is written in Equation (4).

$$GML(\mathbf{k}) = \frac{\mathbf{y}^T\left(\mathbf{I} - \mathbf{H}(\mathbf{k})\right)\mathbf{y}}{\left[\det^+\left(\mathbf{I} - \mathbf{H}(\mathbf{k})\right)\right]^{1/(n-a)}} \tag{4}$$

2.3 Point Estimation of Truncated Spline Nonparametric Regression

A technique for estimating parameter values is known as Maximum Likelihood Estimation (MLE). Adrianingsih & Dani (2021) explain that the MLE method can be used if the distribution of the residual is known. The MLE method is performed by assuming the residuals $\varepsilon_i \sim \text{IIDN}(0,\sigma^2)$. The probability function of ε_i is written in Equation (5) (Setiawan et al., 2017).

$$g(\varepsilon_i) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2\sigma^2}\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}\right) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2\sigma^2}\left(\mathbf{y} - \mathbf{X}(\mathbf{k})\boldsymbol{\lambda}\right)^T\left(\mathbf{y} - \mathbf{X}(\mathbf{k})\boldsymbol{\lambda}\right)\right) \tag{5}$$

Equation (5) is the Normal distribution function of the residuals that forms the basis for forming the likelihood function in Equation (6).

$$L(\boldsymbol{\lambda}) = \prod_{i=1}^{n} g(\varepsilon_i) = \left(2\pi\sigma^2\right)^{-\frac{n}{2}}\exp\left(-\frac{1}{2\sigma^2}\left(\mathbf{y} - \mathbf{X}(\mathbf{k})\boldsymbol{\lambda}\right)^T\left(\mathbf{y} - \mathbf{X}(\mathbf{k})\boldsymbol{\lambda}\right)\right) \tag{6}$$

The likelihood function in Equation (6) forms the basis for parameter estimation under the assumption that the residuals are normally distributed. Then, by performing a logarithmic transformation, Equation (7) obtained.

$$l(\boldsymbol{\lambda}) = \ln L(\boldsymbol{\lambda}) = -\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}(\mathbf{y} - \mathbf{X}(\mathbf{k})\boldsymbol{\lambda})^T(\mathbf{y} - \mathbf{X}(\mathbf{k})\boldsymbol{\lambda}) \tag{7}$$

Next, by using partial derivatives, estimates for the parameters are obtained as in Equation (8).

$$\hat{\boldsymbol{\lambda}} = \left(\mathbf{X}(\mathbf{k})^T\mathbf{X}(\mathbf{k})\right)^{-1}\mathbf{X}(\mathbf{k})^T\mathbf{y} \tag{8}$$

2.4 Interval Estimation of Truncated Spline Nonparametric Regression

Interval estimation is achieved once the pivotal quantity for $\hat{\mathbf{g}}_i(\mathbf{x}), i = 1,2,\ldots,n$ has been determined. The pivotal quantity is formed by transforming as in Equation (9).

$$Q_i(\mathbf{x},\mathbf{y}) = \frac{\hat{g}_i(\mathbf{x}) - E\left(\hat{g}_i(\mathbf{x})\right)}{\sqrt{Var\left(\hat{\mathbf{g}}_i(\mathbf{x})\right)_{ii}}} \tag{9}$$

In the condition where the population variance is unknown, the variance is estimated using the Mean Square Error (MSE) so that Equation (10) is obtained.

$$Q_i(\mathbf{x},\mathbf{y}) = \frac{\hat{g}_i(\mathbf{x}) - g_i(\mathbf{x})}{\sqrt{MSE\left(\mathbf{H}(\mathbf{k})\right)_{ii}}} \tag{10}$$

Then, the pivotal quantity is obtained as shown in Equation (11).

$$Q_i(\mathbf{x},\mathbf{y}) = \frac{\hat{g}_i(\mathbf{x}) - g_i(\mathbf{x})}{\sqrt{\frac{\mathbf{y}^T\left(\mathbf{I} - \mathbf{H}(\mathbf{k})\right)\mathbf{y}}{n-(1+tm+tr)}\left(\mathbf{H}(\mathbf{k})\right)_{ii}}} \tag{11}$$

Equation (11) shows the pivotal quantity used to construct the confidence interval, so that the shortest interval estimation can be constructed as in Equation (12).

$$P\left(\hat{g}_i(\mathbf{x})-t_{\left(\frac{\alpha}{2},(n-(1+tm+tr))\right)}\sqrt{\frac{\mathbf{y}^T(\mathbf{I}-\mathbf{H}(\mathbf{k}))\mathbf{y}}{n-(1+tm+tr)}\mathbf{H}(\mathbf{k})_{ii}}\le g_i(\mathbf{x})\le\hat{g}_i(\mathbf{x})+t_{\left(\frac{\alpha}{2},(n-(1+tm+tr))\right)}\sqrt{\frac{\mathbf{y}^T(\mathbf{I}-\mathbf{H}(\mathbf{k}))\mathbf{y}}{n-(1+tm+tr)}\mathbf{H}(\mathbf{k})_{ii}}\right)=1-\alpha \quad (12)$$

Equation (12) shows the lower and upper limits of the confidence interval containing the true value with confidence level of $1-\alpha$ (Suprapto, 2018).

3. RESULT AND DISCUSSION

3.1 Data Description

Simulated data were generated with sample sizes (n) of 50, 100, 150, 250, and variance (σ^2) of 0.01, 0.25, 0.50, 0.75, 1. Residuals were set to follow a Normal distribution $\varepsilon \sim N(0,\sigma^2)$ with predictor variables x_1 and x_2 following a Uniform distribution $x \sim Unif(0,1)$. The mathematical function used to determine the shape of the regression curve is a trigonometric function written in Equation (15).

$$g(x)=\frac{\sin(3\pi x^5)}{\sin(x^2)}+\frac{\sin(\pi x^6)}{\sin(x^4)} \quad (15)$$

3.2 Scatter Plot

In the process of implementing truncated spline nonparametric regression modeling, the first step is to establish the relationship pattern between the predictor and response variables, which can be illustrated with a scatter plot. For example, scatter plots when the conditions are set to $n=250$ and $\sigma^2=0.01$ are presented in Figure 1.

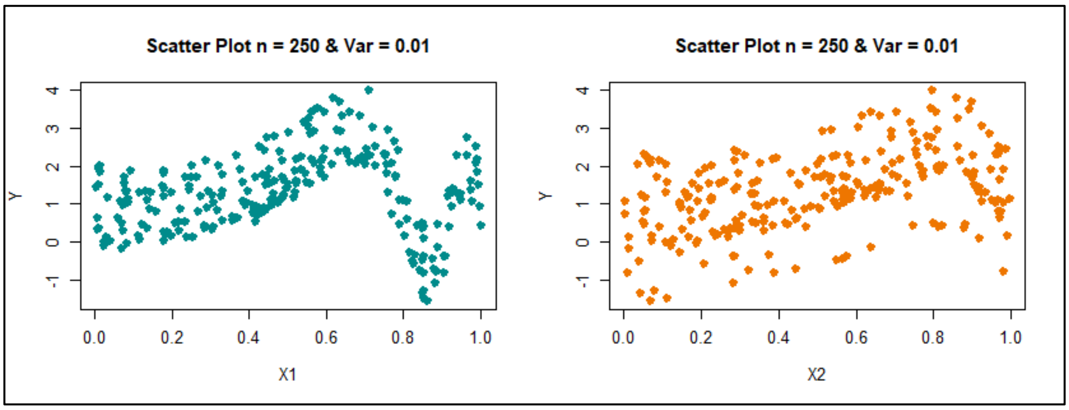


Figure 1. Scatter plot for $n=250$ and $\sigma^2=0.01$

The impact of the simulation can be seen by comparing the scatter plot with other conditions. For example, scatter plots when the conditions are set $n=250$ and $\sigma^2=1$ presented in Figure 2.

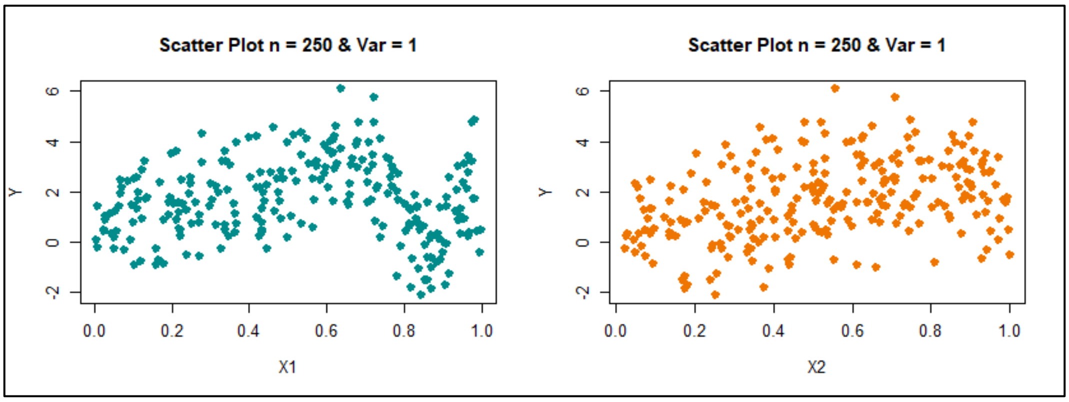


Figure 2. Scatter plot for $n = 250$ and $\sigma^2 = 1$

A comparison of Figures 1 and 2 shows that increasing variance results in a wider dispersion of data points. Additionally, the observed data pattern is consistent with the properties of the truncated spline estimator, which varies at specific sub-intervals.

3.3 Simulation Study

This simulation study aimed to assess the performance of the truncated spline estimator under various predetermined data conditions. Simulation data were generated based on several characteristics, including sample size, variance, and knot points. The simulation was conducted over 10 iterations with the performance of the method evaluated based on changes in the average GML, R^2, dan MSE values. After conducting the simulation using a nonparametric truncated spline regression model with 1, 2, and 3 knot points, the results were obtained and presented in Table 1, Table 2, and Table 3.

Table 1. Simulation Results using 1 Knot Point

Sample Size	Criteria	Variance				
		0.01	**0.25**	**0.50**	**0.75**	**1**
50	GML	19.9058	25.8147	31.8263	38.0793	43.4248
	R^2	0.5237	0.4640	0.4042	0.3471	0.3255
	MSE	0.6044	0.7773	0.9878	1.2286	1.4021
100	GML	3.2268	4.4805	5.7430	6.8468	8.1067
	R^2	0.5617	0.4633	0.3712	0.2964	0.2686
	MSE	0.5505	0.7823	1.0560	1.3277	1.5714
150	GML	1.7262	2.4370	3.1500	3.7972	4.5443
	R^2	0.6115	0.5351	0.4752	0.4337	0.3836
	MSE	0.5370	0.7580	0.9808	1.1827	1.4372
250	GML	1.0276	1.4710	1.9391	2.4209	2.8761
	R^2	0.5837	0.4956	0.4277	0.3721	0.3358
	MSE	0.5176	0.7407	0.9763	1.2187	1.4496

Table 2. Simulation Results using 2 Knot Point

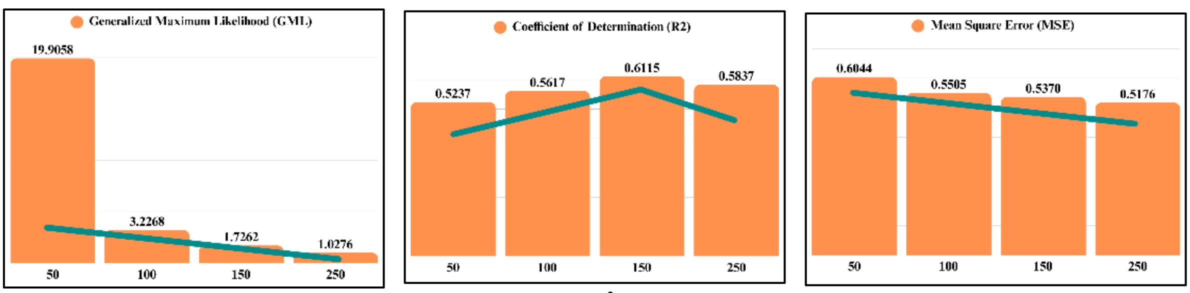
Sample Size	Criteria	Variance				
		0.01	**0.25**	**0.50**	**0.75**	**1**
50	GML	25.6811	65.3969	92.3335	115.7423	137.0197
	R^2	0.9095	0.7119	0.5491	0.4907	0.4148
	MSE	0.1118	0.4161	0.7628	0.9757	1.2336
100	GML	1.3982	4.1614	6.9688	9.4719	11.9385
	R^2	0.9096	0.7708	0.6530	0.5246	0.4123
	MSE	0.1120	0.3335	0.5854	0.8983	1.2568
150	GML	0.6181	1.6717	2.7612	3.7931	4.8990
	R^2	0.9129	0.7992	0.7109	0.6416	0.5845
	MSE	0.1192	0.3261	0.5386	0.7460	0.9670
250	GML	0.3281	0.9163	1.5295	2.1351	2.7530
	R^2	0.8981	0.7599	0.6557	0.5775	0.5061
	MSE	0.1256	0.3506	0.5850	0.8174	1.0756

Table 3. Simulation Results using 3 Knot Point

Sample Size	Criteria	Variance				
		0.01	**0.25**	**0.50**	**0.75**	**1**
50	GML	57.4934	195.7574	291.8006	365.9275	442.4757
	R^2	0.9590	0.7220	0.6438	0.5235	0.4839
	MSE	0.0509	0.4017	0.5988	0.8953	1.0698
100	GML	1.2490	6.1897	10.6881	14.8556	19.1152
	R^2	0.9606	0.7930	0.6833	0.5912	0.5223
	MSE	0.0480	0.3002	0.5328	0.7750	1.0308
150	GML	0.4623	2.0447	3.5482	4.9109	6.4057
	R^2	0.9585	0.8349	0.7366	0.6548	0.5944
	MSE	0.0574	0.2680	0.4908	0.7199	0.9459
250	GML	0.1884	0.9233	1.6583	2.3747	3.0834
	R^2	0.9549	0.8077	0.6931	0.6051	0.5387
	MSE	0.0555	0.2806	0.5204	0.7631	1.0026

3.1.1 Simulation Based on Sample Size

Sample size is one of the important factors that can affect the validity of simulation results. The impact of varying sample sizes is visualized in Figure 3, where the variance value is set at 0.01.



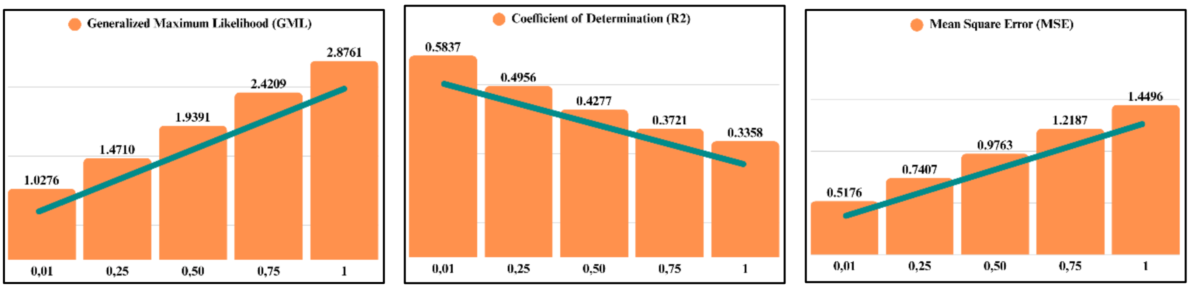
(a) GML value (b) R^2 value (c) MSE value

Figure 3. Visualization of the impact of sample size

Figure 3 shows that the sample size affects the performance of the truncated spline. The larger the sample size, the GML and MSE values tend to decrease, reflecting more efficient knot point selection and smaller prediction errors. The R^2 value generally increases, indicating a better model fit, although a slight decrease occurs at the sample size of 250, possibly due to noise interference in the larger data

3.1.2 Simulation Based on Variance

The impact of different variances on the performance of truncated spline nonparametric regression can be visualized in Figure 4, where the sample size is set at 250.



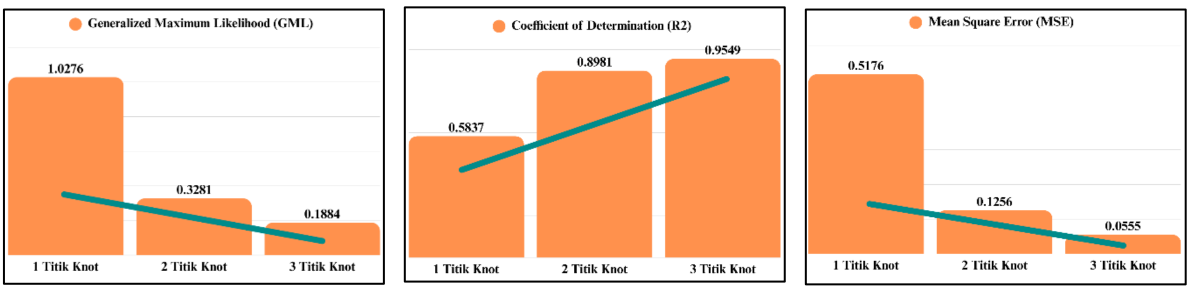
(a) GML value (b) R^2 value (c) MSE value

Figure 4. Visualization of the impact of variance

The simulation results in Figure 4 show that data variance affects the performance of the truncated spline. When the variance increases, the GML and MSE values increase, indicating an increase in model complexity and the magnitude of prediction errors. Meanwhile, the decreasing value of R^2 indicates that the model's ability to explain data variation is diminishing.

3.1.3 Simulation Based on Knot Points

The impact of knot points is visualized in Figure 5, where the sample size is set at 250 and the variance is set at 0.01.



(a) GML value (b) R^2 value (c) MSE value

Figure 5. Visualization of the impact of knot points

Figure 5 shows that the number of knot points affects the performance of the truncated spline. As the number of knot points increases, the GML and MSE values tend to decrease, indicating the model becomes more flexible and accurate in capturing data patterns. The increasing R^2 value suggests an improved model fit. The addition of more knot points makes the model more flexible in adjusting to data patterns, but it should be noted that too many knot points can increase the risk of overfitting, so selecting the optimal number of knot points is a crucial step in the truncated spline nonparametric regression.

3.4 Interval Estimation

According to the simulation results, the truncated spline nonparametric regression model with 3 knot points exhibits the smallest GML value, so this model will be used in the interval estimation process for parameters and the regression curve. For example, the interval estimation results for parameters and the regression curve under the condition of $n=50$ with $\sigma^2=0.01$ are shown in Table 4 and Table 5.

Table 4. Parameter Interval Estimation for $n=50$ and $\sigma^2=0.01$

Variable	Parameter	Parameter Point Estimation	Parameter Interval Estimation	
			Lower Limit	Upper Limit
Constant	$\hat{\lambda}_0$	-0.0969	-0.8872	0.6933
x_1	$\hat{\lambda}_{11}$	1.4588	0.2836	2.6339
	$\hat{\lambda}_{12}$	2.0919	1.1004	3.0835
	$\hat{\lambda}_{13}$	-134.6485	-182.5210	-86.7759
	$\hat{\lambda}_{14}$	293.9488	180.4334	407.4642
x_2	$\hat{\lambda}_{21}$	-214.6621	-331.8991	-97.4250
	$\hat{\lambda}_{22}$	8.8144	-18.6430	36.2717
	$\hat{\lambda}_{23}$	-32.6829	-95.9357	30.5699
	$\hat{\lambda}_{24}$	-28.7695	-124.9665	67.4274

Table 5. Regression Curve Interval Estimation for $n=50$ and $\sigma^2=0.01$

Actual Regression Curve	Regression Curve Point Estimation	Regression Curve Interval Estimation	
		Lower Limit	Upper Limit
2.6024	2.5178	1.9960	3.0396
2.1743	2.0722	1.6108	2.5337
0.6311	1.0629	0.7179	1.4078
0.9615	1.2464	0.9429	1.5500
⋮	⋮	⋮	⋮
0.1882	0.2401	-0.3717	0.8520

Subsequently, a comparison is made between the point estimates and the interval estimates to assess whether the point estimate falls within the interval estimate range. The comparison results are visualized in Figure 6.

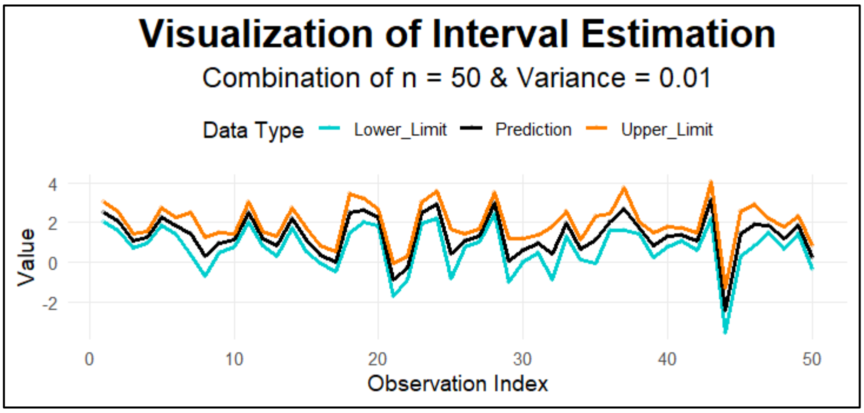


Figure 6. Visualization of interval estimation for $n = 50$ and $\sigma^2 = 0.01$

The same procedure was applied to each combination of sample size and variance. Another example of interval estimation under different conditions is shown in Figure 7.

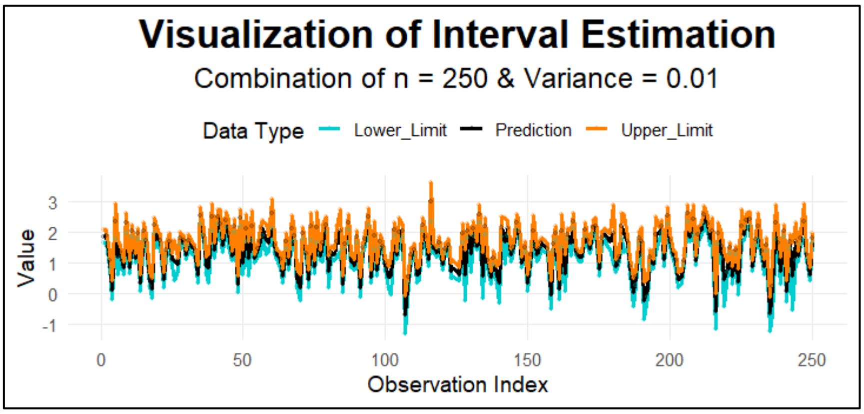


Figure 7. Visualization interval estimation for $n = 250$ and $\sigma^2 = 0.01$

Figure 6 and Figure 7 show that the point estimates consistently fall within the interval bounds, indicating the model's ability to capture underlying data patterns. As the sample size increases, the narrower the interval becomes, which means the estimation error decreases and precision increases. Conversely, with high variance, the interval widens due to increased uncertainty from noise, indicating that the model is quite sensitive to data disturbances. Overall, the results of this study indicate that data characteristics such as the sample size, variance, and knot points significantly affect the accuracy and precision of the performance of the truncated spline estimator, including its interval estimation results. These findings can serve as a reference for applied research in determining the appropriate data conditions for applying truncated spline nonparametric regression to obtain a more optimal model and reliable estimates.

4. CONCLUSION

Based on the results of the analysis and discussion, conclusions were drawn by the research objectives. The simulation results demonstrate the impact of varying sample sizes on the performance of the truncated spline. Specifically, as the sample size increases, the GML and MSE values decrease while R^2 increases. This indicates that with larger datasets, the model becomes more efficient in determining the optimal number of knot points and is better able to explain the overall variation in the data, thereby reducing prediction errors.

The impact of increased variance is that the GML and MSE values increase while R^2 decreases. The increase in GML value indicates that the model becomes more complex in determining the optimal knot points, but the decrease in R^2 value and the increase in MSE value indicate that the model becomes less capable of explaining the more dispersed data variation and less effective in producing accurate predictions.

The truncated spline nonparametric regression model achieves optimal performance with 3 knots. The impact of the knots on the performance of the truncated spline is that when the number of knots is increased, the GML and MSE values decrease, while the R^2 value increases. This indicates that a model with more knots is more flexible in adapting to complex data patterns, thereby better explaining the variability in the data and providing predictions with higher accuracy.

5. RECOMMENDATION

This study is limited to the use of only 1, 2, and 3 knot points. Therefore, future studies are recommended to explore a combination of knot points to obtain more accurate and flexible estimation results in representing the relationship patterns between variables.

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