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Optimization of LPG Distribution on VRP Using Cheapest Insertion Heuristics Algorithms and Dynamic Programming

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Abstract
The optimization of distribution routes is frequently limited, especially for LPG agents of PT. Cahaya Mentari Bumi Perkasa, as routes are predominantly established based on driver intuition. This agency operates 35 bases with a fleet of 3 trucks, each capable of carrying 560 cylinders. This study seeks to identify the most efficient distribution route and analyze the cost differentials between the original route and the suggested route utilizing the Vehicle Routing Problem (VRP) model, specifically the Capacitated Vehicle Routing Problem (CVRP), which incorporates capacity and fluctuating demand limitations. A quantitative methodology utilizing secondary data was implemented, encompassing demand statistics per base, vehicle capacity, and inter-location distances. The Cheapest Insertion Heuristics (CIH) technique serves as the initial solution, while Dynamic Programming (DP) functions as the precise approach for optimal route refining. The findings indicate that the amalgamation of DP and CIH yields a more effective distribution path, achieving a 28% decrease in travel distance, an 8.2% decrease in journey time, and a 19.9% reduction in distribution expenses. The quantity of delivery tours diminished from 9 to 8, enhancing fleet efficiency and the efficacy of the company's distribution.

Keywords: Cheapest Insertion Heuristics Algorithm; Dynamic Programming; VRP

Abstrak
Penentuan rute optimal dalam distribusi sering terkendala, khususnya pada agen LPG PT. Cahaya Mentari Bumi Perkasa, karena rute masih ditentukan berdasarkan intuisi supir. Agen ini melayani 35 pangkalan dengan 3 armada truk berkapasitas 560 tabung. Penelitian ini bertujuan untuk menentukan rute distribusi optimal dan membandingkan biaya distribusi antara rute awal dan rute usulan menggunakan model *Vehicle Routing Problem* (VRP) dengan kendala kapasitas dan permintaan bervariasi, yakni *Capacitated Vehicle Routing Problem* (CVRP). Pendekatan kuantitatif dengan data sekunder digunakan, mencakup data permintaan per pangkalan, kapasitas kendaraan, dan jarak antarlokasi. Algoritma *Cheapest Insertion Heuristics* (CIH) digunakan sebagai solusi awal, dan *Dynamic Programming* (DP) sebagai algoritma eksak untuk penyempurnaan rute secara optimal. Hasil menunjukkan bahwa integrasi DP dan CIH menghasilkan rute distribusi yang lebih efisien, dengan penghematan jarak tempuh 28%, waktu tempuh 8,2%, dan biaya distribusi 19,9%. Jumlah tur perjalanan juga berkurang dari 9 menjadi 8 sehingga meningkatkan efisiensi armada dan efektivitas distribusi perusahaan.

Kata Kunci: Cheapest Insertion Heuristics Algorithm; Dynamic Programming; VRP

1. INTRODUCTION

1.1 Research Background

Distribution is an essential function in corporate operations that necessitates an efficient route planning system to minimize operating expenses and travel time, particularly in the distribution of commodities from producers to consumers (Ardiansyah et al., 2024). The efficacy of distribution is significantly impacted by the choice of the appropriate route. Inefficient routes might augment journey distance, elevate travel costs, and diminish client satisfaction. Consequently, it is essential to implement measures that minimize costs, distance, and delivery time to enhance efficiency and profitability (Tegar, 2019).

PT. Cahaya Mentari Bumi Perkasa is a firm involved in the distribution of subsidized LPG gas in Medan, situated in Jl. Purwosari No. 122, Medan Timur. Companies frequently encounter obstacles in gas delivery to bases, such as restricted fleet size, insufficient truck capacity, and suboptimal distribution routes that depend on driver intuition. Consequently, a distribution system is required that can address these problems via effective route optimization.

1.2 Literature Review

The issue of LPG distribution routes can be represented through the Vehicle Routing Problem (VRP), a mathematical framework for coordinating supplies from a singular depot to several consumers and returning to the depot (Fikri Akbar L & Rosnani Ginting, 2019). The Capacitated Vehicle Routing Problem (CVRP) model is employed in LPG distribution to address limited fleet capacity and fluctuating customer demand, with the objective of minimizing distribution distance and costs while optimizing vehicle capacity usage (Adhi et al., 2019),(Chandra & Setiawan, 2018).

Diverse methodologies have been employed to solve the VRP or CVRP, including the Cheapest Insertion Heuristic (CIH) algorithm, which incrementally constructs routes by incorporating points with the minimal distance (Rizki Putra Sinaga & Faridawaty Marpaung, 2023). This method, while rapid and simple to execute, does not consistently yield optimal results (Saleh, Helmi and Prihandono, 2015). Consequently, heuristic algorithms are frequently integrated with precise methodologies like Dynamic Programming (DP), which methodically addresses problems incrementally to derive the optimal solution (Gunawan & Andriani, 2023). Some previous studies have employed heuristic methods to address the Vehicle Routing Problem (VRP). Studies demonstrate that heuristic techniques, including Nearest Neighbor, Saving Method, and Cheapest Insertion Heuristics (CIH), can produce rapid solutions for the formulation of distribution routes (Waridah & Madja, 2023), (Kendal, 2022), (Indrianti et al., 2025) (Yudhi, Syarifah Ratih Eka Wahyuni, 2020). Nevertheless, the majority of these studies employed a singular heuristic algorithm without amalgamating it with alternative optimization techniques, leading to solutions that are often near-optimal and may not be appropriate for distribution scenarios with intricate capacity constraints, exemplified by the Capacitated Vehicle Routing Problem (CVRP) in LPG distribution. Several studies have endeavored to amalgamate the CIH algorithm with Dynamic Programming (DP) to address the Traveling Salesman Problem (TSP) (Akmal, 2018), (Khadafi & Saputri, 2023),(Utomo et al., 2018). This approach, while intriguing, is confined to the TSP, which overlooks demand fluctuations and vehicle capacity, rendering it less applicable to real-world distribution scenarios like LPG distribution with a constrained fleet. Furthermore, research integrates CIH with the Ant Colony algorithms (Soenandi et al., 2019) and compares CIH with other algorithms, including Greedy and Dynamic Programming (Hignasari, 2020),(Aristi, 2015). This technique prioritizes the comparison of algorithm performance over practical application in scenarios involving distribution with capacity limitations and geographical constraints. This study highlights key gaps in previous research. Few studies have combined heuristic algorithms like Cheapest Insertion Heuristic (CIH) with exact methods such as Dynamic Programming (DP) to solve the Capacitated Vehicle Routing Problem (CVRP) in the context of subsidized LPG distribution, which involves fluctuating demand and limited fleet capacity. Most past research focused on general routing problems (TSP or VRP) without considering these constraints. Moreover, no prior studies have applied these methods in real distribution companies like PT. Cahaya Mentari Bumi Perkasa. This study addresses these gaps with a practical, context-specific approach.

This research seeks to develop and execute a methodology for addressing the Capacitated Vehicle Routing Problem (CVRP) that combines the Cheapest Insertion Heuristic algorithm with Dynamic Programming to ascertain the optimal LPG gas distribution route, and to evaluate the efficacy of the proposed route in comparison to the company's current route regarding distance traveled, travel time, and distribution costs. This research introduces a hybrid methodology that merges the rapid solution capabilities of the Cheapest Insertion Heuristic algorithm with the optimization precision of Dynamic Programming, a combination that has not been extensively utilized in the context of LPG distribution with constrained fleet size and fluctuating demand in urban environments such as Medan. This work diverges from prior research, which was often broad or focused on a single method, by adapting the algorithmic approach to real-world spatial and operational settings, thereby offering both practical and academic advancements in the development of applied CVRP solutions.

2. RESEARCH METHOD

This study employed quantitative methodologies and was conducted at PT. Cahaya Mentari Bumi Perkasa, on Jl. Purwosari No. 122, Pulo Brayan Bengkel District, East Medan District, Medan City.

2.1 Data Types and Sources

Secondary data categories were acquired from existing company records, including base locations, demand per base, fleet type utilized, fleet capacity (vehicles), and time.

2.2 Vehicle Routing Problem

The Vehicle Routing Problem (VRP) is a methodology for optimizing vehicle routes to fulfill client demands as efficiently as possible according to specified criteria (Prasetyo & Tamyiz, 2017). The VRP was designed to reflect real-world situations. The CVRP model is a variant of the VRP that addresses challenges associated with vehicle capacity constraints, assuming symmetrical distances $Cij = Cji$. Defined: for each $E = \{(i,j) \mid i,j \in V, (i \neq j)\}$. (i,j) the edge from point i to j, starting at 0 and concluding at 0. The set k represents a vehicle with a load capacity of q. Each agent i, for all $i \in V$, possesses a request, so constraining the route length by the vehicle's capacity, which is expressed as:

$$x_{ijk} : x_{ijk} = \begin{cases} 1, \text{ if a trip is found from } i \text{ to } j \text{ using vehicle } k \text{ to } 0 \\ 0, \text{If no trip exists from } i \text{ to } j \text{ with vehicle } k \text{ to } 0 \end{cases}$$

2.3 Cheapest Insertion Heuristics

The Cheapest Insertion Heuristic was methodologically chosen for its suitability in solving the Vehicle Routing Problem (VRP), offering near-optimal solutions with low computational time. Its incremental structure allows adaptation to dynamic LPG distribution and the addition of delivery points, while its flexibility to constraints like vehicle capacity and delivery time makes it practical for real-world applications.

2.4 Dynamic Programming

Dynamic Programming enhances the Cheapest Insertion Heuristic by solving subproblems efficiently and reducing recalculations through tabulation.

2.5 Research Design

The research design steps to ascertain the appropriate LPG distribution route at PT. Cahaya Mentari Bumi Perkasa are as follows:

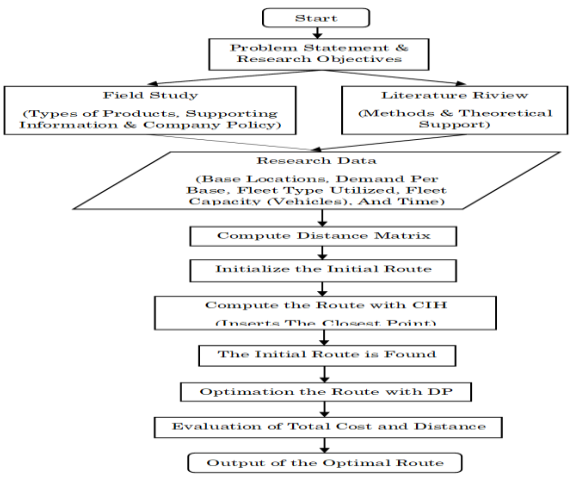

Figure 1. Flowchart

2.6 Data Processing Tools

a. Microsoft Excel : To compute the distance matrix and demand of base
b. Python : To implementation of heuristic algorithms
c. ArcGis : To represent the path in a picture
d. Google Map : To find out the latitude and longitude coordinates of each base.

3. RESULTS AND DISCUSSION

3.1 Collection of Data

Each of the 35 active LPG distribution stations at PT. Cahaya Mentari Bumi Perkasa has its own address and demand for LPG. The subsequent list comprises stations and their respective demand:

Table 1. Demand Data

No	Gas Station	Location	Demand (Cylinders)
1	Naga Mas	Jl. Bawal No 7a Lk XIX, Kel. Belawan Bahagia	62
2	Oke Gas	Jl. Bromo Ujung Gg Rezeki No.03, Kel. Binjai	141
3	Ikram	Jl. Seibahbolon No. 25 E, Kel. Babura	70
4	Tangkas S.	Jl. Boxit Perjuangan Lk I,Kel. Kota Bangun	140
5	Edi Joyo	Jl. Mangaan 3 Lk 9, Kel. Mabar	153
6	Esrawati A. Sianipar	Jl. Bunga Lau No. 12/60 Lingkungan Iii, Kel. Kemenangan Tani	59
7	Aril Tarigan	Jl. Bunga Rampe Raya No. 54, Kel. Simalingkar B	75
8	Keluarga Sikumbang	Jl. Megawati Gg. Damai No. 23, Kel. Pasar Merah Timur	140
9	Zulfa	Jl. Gedung Arca, Gang Jawa, Kel. Pasar Merah Timur	147
10	Salam Gurusinga	Jl. Bendungan I No. 18, Kel. Bangun Mulia	139
11	Zhafira Elpiji	Jl. Selamat Pulau No. 4a, Kel. Sitirejo Iii	70
12	Ragil	Jl. Setia Luhur Lk Vi No.103, Kel. Helvetia	127
13	Serasi	Jl. Persatuan No 36-A, Kel. Helvetia Timur	96
14	Mubarrak	Jl. Pertiwi, Kel. Bantan	114
15	Ahmadsyah	Jl. Tirto Sari Gg. Amal No. 84, Kel. Bantan	128
16	Irham Berkah	Jl. Tangguk Damai No. 32 Blok I Griya Martubung, Kel. Besar	161
17	Taruna Gas	Jl. Rawe 1 Lingk. 12, Kel. Tangkahan	85
18	Suratman Dj	Jl. Starban Gang Lurah No. 42, Kel. Polonia	70
19	Aqilla Qirana	Gg. Wakaf Ii, Kel. Lalang	152
20	Rajawali Gas	Jl. Gaharu Gg. Amat Lama, Kel. Gaharu	70
21	Sam Gas	Jl. Pasar Iii Gg. Melur No. 10, Kel. Glugur Darat I	55
22	Fatmawati Chan	Jl. 3 No. B-49 Link. X, Kel. Pb Bengkel Baru	119
23	Kembar Pakpahan	Jl. Tempuling No. 73, Kel. Sidorejo	140
24	Rizqa	Jl. Purwosari No.120, Kel. Pb Bengkel Baru	70
25	Ara Kana Ad	Jl. Miring No. B-3, Kel. Pb Bengkel Baru	183
26	Pangkalan Irfan	Komp Pln Paya Pasir Jl Pltu No 20 B Lk 33, Kel. Rengas Pulau	134
27	Faisal	Jl. Marelan Raya, Gang Manggis, Kel. Tanah Enam Ratus	67
28	Toko Rizky	Jalan Pasar 1 Rel No. 2, Kel. Terjun	140
29	Nurlitasari P.	Jl. Aksara No. 69, Kel. Pahlawan	82
30	Polman Simbolon	Jl. M Taufik Gg. Tunggal No.22 Medan, Kel. Tegalrejo	120
31	Adamdanish	Jl Karya Gg Salak No. 10a, Kel. Karang Berombak	131
32	Ganda	Jl Mesjid Gg Jaya No 39 B, Kel. Sei Agul	84
33	Zainal Efendy	Jl. Utama No. 7-1 A, Kel. Kota Matsum Iii	178
34	Hilman Situmorang	Jl. Turi No. 175, Kel. Teladan Timur	117
35	Mukhlis	Jl. Flamboyan Raya Gg. Bersama, Kel. Tanjungsari	70

Source: Data from the Company

PT. Cahaya Mentari Bumi Perkasa operates three fleets, including a Canter truck with a capacity of 560 cylinders. Consequently, distribution requires several days to fulfill all fundamental necessities(Soimun et al., 2025).

Table 2. Specifications of the Transport Armada

Vehicle	Capacity (Cylinders)	Specifications (*mm*)	***Gross Vehicle Weight (ton)***
Truk (*Canter*) FE 73 HD	560	Length (4.200) Width (1.870) Height (2.130)	7500

3.2 Coordinate Points

Utilizing the base address data, two geographic coordinates will be derived: latitude and longitude on Google Maps(Singal & Rindengan, 2021).

Table 3. Coordinate Points

No	Gas Station	Node	Lattitude (x)	Longitude (y)	No	Gas Station	Node	Lattitude (x)	Longitude (y)
1	Cahaya M.B.Perkasa	Z	3,63318	98,69139	19	Suratman Dj	A18	3,55513	98,66567
2	Naga Mas	A1	3,76768	98,67585	20	Aqilla Qirana	A19	3,59095	98,60785
3	Oke Gas	A2	3,56889	98,71197	21	Rajawali Gas	A20	3,60438	98,67764
4	Ikram	A3	3,58348	98,65827	22	Sam Gas	A21	3,61602	98,68481
5	Tangkas Siahaan	A4	3,67463	98,66249	23	Fatmawati Chan	A22	3,63225	98,68055
6	Edi Joyo	A5	3,65764	98,6768	24	Kembar Pakpahan	A23	3,60749	98,69586
7	E. A. Sianipar	A6	3,51995	98,61012	25	Rizqa	A24	3,63311	98,69137
8	Aril Tarigan	A7	3,49785	98,64462	26	Ara Kana Ad	A25	3,63408	98,67415
9	K. Sikumbang	A8	3,57157	98,70092	27	P. Irfan	A26	3,71261	98,6656
10	Zulfa	A9	3,57066	98,69712	28	Faisal	A27	3,68261	98,65457
11	Salam Gurusinga	A10	3,53337	98,7407	29	Toko Rizky	A28	3,68943	98,64733
12	Zhafira Elpiji	A11	3,54933	98,70009	30	Nurlitasari P.	A29	3,59228	98,70589
13	Ragil	A12	3,60127	98,63657	31	Polman Simbolon	A30	3,60577	98,69447
14	Serasi	A13	3,61227	98,65147	32	Adamdanish	A31	3,61457	98,66438
15	Mubarrak	A14	3,59339	98,73308	33	Ganda	A32	3,60529	98,66159
16	Ahmadsyah	A15	3,59219	98,72561	34	Zainal Efendy	A33	3,5745	98,68916
17	Irham Berkah	A16	3,69943	98,68579	35	Hilman Stmrg	A34	3,56303	98,7105
18	Taruna Gas	A17	3,697	98,69846	36	Mukhlis	A35	3,54139	98,62026

3.3 Distance Matrix Calculation

The base distance computation use the haversine formula(Putra et al., 2015). The computation involves determining the distance between two places on the Earth's surface by measuring the straight-line distance while accounting for the Earth's curvature, utilizing trigonometric principles (Miftahuddin et al., 2020). These two points are derived from the latitude and longitude of each base(Abadi Nugroho, 2020). With:

$$d = 2r \text{ . arc sin } (\sqrt{a})$$

$$d = 2r \text{ . arc sin } \left(\sqrt{\sin^2\left(\frac{\Delta lat}{2}\right) + \cos(lat_1).\cos(lat_2).\sin^2\left(\frac{\Delta long}{2}\right)} \right) \quad (1)$$

Details: d = distance; Δlat = significant change in latitude

r = radius of earth (6,731 km); $\Delta long$ = significant change in longitude

The coordinate values are transformed into radian form:

a. Lattitude (Z) = 3,63318→0,06342

Lattitude (A1) =3,76768→0,06576

b. Longitude (Z) = 98,69139→1,72271

Longitude (A1) = 98,67585→1,72244

	Z	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17
Z	0	15,1	7,5	6,6	5,6	3,2	15,5	15,9	6,9	7	12,4	9,4	7,1	5	6,4	5,9	7,4	7,1
A1	15,1	0	22,5	20,6	10,5	12,3	28,5	30,2	22	22,1	27,1	24,5	19	17,5	20,4	20,3	7,7	8,3
A2	7,5	22,5	0	6,2	13	10,6	12,6	10,9	1,3	1,7	5,1	2,5	9,1	8,3	3,6	3	14,8	14,3
A3	6,6	20,6	6,2	0	10,2	8,5	8,9	9,7	4,9	4,6	10,7	6	3,1	3,3	8,4	7,6	13,3	13,4
A4	5,6	10,5	13	10,2	0	2,5	18,2	19,8	12,2	12,2	18	14,6	8,7	7	12	11,6	3,8	4,7
A5	3,2	12,3	10,6	8,5	2,5	0	17	18,1	10	9,9	15,6	12,3	7,7	5,8	9,5	9,1	4,8	5
A6	15,5	28,5	12,6	8,9	18,2	17	0	4,6	11,6	11,2	14,6	10,5	9,5	11,3	15,9	15,2	21,7	22
A7	15,9	30,2	10,9	9,7	19,8	18,1	4,6	0	10,3	10	11,4	8,4	11,5	12,8	14,5	13,8	22,9	23
A8	6,9	22	1,3	4,9	12,2	10	11,6	10,3	0	0,4	6,1	2,5	7,9	7,1	4,3	3,6	14,3	14
A9	7	22,1	1,7	4,6	12,2	9,9	11,2	10	0,4	0	6,4	2,4	7,6	6,9	4,7	4	14,4	14,1
A10	12,4	27,1	5,1	10,7	18	15,6	14,6	11,4	6,1	6,4	0	4,9	13,8	13,3	6,7	6,8	19,5	18,8
A11	9,4	24,5	2,5	6	14,6	12,3	10,5	8,4	2,5	2,4	4,9	0	9,1	8,9	6,1	5,6	16,8	16,4
A12	7,1	19	9,1	3,1	8,7	7,7	9,5	11,5	7,9	7,6	13,8	9,1	0	2,1	10,8	10	12,2	12,7
A13	5	17,5	8,3	3,3	7	5,8	11,3	12,8	7,1	6,9	13,3	8,9	2,1	0	9,3	8,6	10,4	10,8
A14	6,4	20,4	3,6	8,4	12	9,5	15,9	14,5	4,3	4,7	6,7	6,1	10,8	9,3	0	0,8	12,9	12,2
A15	5,9	20,3	3	7,6	11,6	9,1	15,2	13,8	3,6	4	6,8	5,6	10	8,6	0,8	0	12,7	12,1
A16	7,4	7,7	14,8	13,3	3,8	4,8	21,7	22,9	14,3	14,4	19,5	16,8	12,2	10,4	12,9	12,7	0	1,4
A17	7,1	8,3	14,3	13,4	4,7	5	22	23	14	14,1	18,8	16,4	12,7	10,8	12,2	12,1	1,4	0
A18	9,1	23,7	5,4	3,3	13,3	11,5	7,3	6,8	4,3	3,9	8,7	3,9	6,1	6,6	8,6	7,8	16,2	16,2
A19	10,4	21,1	11,8	5,7	11,1	10,7	7,9	11,1	10,6	10,2	16,1	11,3	3,4	5,4	13,9	13,1	14,9	15,5
A20	3,6	18,2	5,5	3,2	8	5,9	12	12,4	4,5	4,3	10,6	6,6	4,6	3	6,3	5,5	10,6	10,6
A21	2	16,9	6,1	4,7	7	4,7	13,5	13,9	5,3	5,2	11,1	7,6	5,6	3,7	5,9	5,3	9,3	9,1
A22	1,2	15,1	7,9	6	5,1	2,9	14,8	15,5	7,1	7,1	12,9	9,5	6	3,9	7,3	6,7	7,5	7,5
A23	2,9	18	4,7	5	8,3	6	13,6	13,5	4	4,1	9,6	6,5	6,6	5	4,4	3,7	10,3	10
A24	0,1	15,1	7,5	6,6	5,6	3,2	15,5	15,9	6,9	7	12,4	9,4	7,1	5	6,4	5,9	7,4	7,2
A25	1,9	14,9	8,4	5,9	4,7	2,6	14,6	15,5	7,6	7,5	13,4	9,9	5,6	3,5	8	7,4	7,4	7,5
A26	9,3	6,2	16,8	14,4	4,2	6,2	22,3	24	16,2	16,2	21,6	18,6	12,8	11,3	15,2	15	2,7	4
A27	6,9	9,8	14,2	11	1,3	3,7	18,8	20,6	13,4	13,3	19,2	15,7	9,3	7,8	13,2	12,8	3,9	5,1
A28	8	9,3	15,2	11,9	2,4	4,8	19,3	21,3	14,4	14,3	20,2	16,7	9,9	8,6	14,3	13,9	4,4	5,8
A29	4,8	19,8	2,7	5,4	10,4	8	13,4	12,5	2,4	2,6	7,6	4,8	7,8	6,5	3	2,2	12,1	11,7
A30	3,1	18,1	4,5	4,7	8,5	6,1	13,4	13,2	3,9	3,9	9,6	6,3	6,5	4,8	4,5	3,8	10,5	10,2
A31	3,7	17,1	7,3	3,5	6,7	5	12,1	13,2	6,3	6,1	12,4	8,3	3,4	1,5	8	7,3	9,7	9,9
A32	4,5	18,1	6,9	2,5	7,7	6,1	11,1	12,1	5,8	5,5	11,9	7,6	2,8	1,4	8,1	7,3	10,8	11
A33	6,5	21,6	2,6	3,6	11,5	9,4	10,7	9,9	1,3	1	7,3	3,1	6,6	5,9	5,3	4,5	13,9	13,7
A34	8,1	23,1	0,7	6,2	13,5	11,2	12,2	10,3	1,4	1,7	4,7	1,9	9,3	8,6	4,2	3,7	15,4	15
A35	12,9	25,9	10,7	6,3	15,6	14,4	2,6	5,6	9,6	9,2	13,4	8,9	6,9	8,6	13,8	13	19	19,4

	A18	A19	A20	A21	A22	A23	A24	A25	A26	A27	A28	A29	A30	A31	A32	A33	A34	A35
Z	9,1	10,4	3,6	2	1,2	2,9	0,1	1,9	9,3	6,9	8	4,8	3,1	3,7	4,5	6,5	8,1	12,9
A1	23,7	21,1	18,2	16,9	15,1	18	15,1	14,9	6,2	9,8	9,3	19,8	18,1	17,1	18,1	21,6	23,1	25,9
A2	5,4	11,8	5,5	6,1	7,9	4,7	7,5	8,4	16,8	14,2	15,2	2,7	4,5	7,3	6,9	2,6	0,7	10,7
A3	3,3	5,7	3,2	4,7	6	5	6,6	5,9	14,4	11	11,9	5,4	4,7	3,5	2,5	3,6	6,2	6,3
A4	13,3	11,1	8	7	5,1	8,3	5,6	4,7	4,2	1,3	2,4	10,4	8,5	6,7	7,7	11,5	13,5	15,6
A5	11,5	10,7	5,9	4,7	2,9	6	3,2	2,6	6,2	3,7	4,8	8	6,1	5	6,1	9,4	11,2	14,4
A6	7,3	7,9	12	13,5	14,8	13,6	15,5	14,6	22,3	18,8	19,3	13,4	13,4	12,1	11,1	10,7	12,2	2,6
A7	6,8	11,1	12,4	13,9	15,5	13,5	15,9	15,5	24	20,6	21,3	12,5	13,2	13,2	12,1	9,9	10,3	5,6
A8	4,3	10,6	4,5	5,3	7,1	4	6,9	7,6	16,2	13,4	14,4	2,4	3,9	6,3	5,8	1,3	1,4	9,6
A9	3,9	10,2	4,3	5,2	7,1	4,1	7	7,5	16,2	13,3	14,3	2,6	3,9	6,1	5,5	1	1,7	9,2
A10	8,7	16,1	10,6	11,1	12,9	9,6	12,4	13,4	21,6	19,2	20,2	7,6	9,6	12,4	11,9	7,3	4,7	13,4
A11	3,9	11,3	6,6	7,6	9,5	6,5	9,4	9,9	18,6	15,7	16,7	4,8	6,3	8,3	7,6	3,1	1,9	8,9
A12	6,1	3,4	4,6	5,6	6	6,6	7,1	5,6	12,8	9,3	9,9	7,8	6,5	3,4	2,8	6,6	9,3	6,9
A13	6,6	5,4	3	3,7	3,9	5	5	3,5	11,3	7,8	8,6	6,5	4,8	1,5	1,4	5,9	8,6	8,6
A14	8,6	13,9	6,3	5,9	7,3	4,4	6,4	8	15,2	13,2	14,3	3	4,5	8	8,1	5,3	4,2	13,8
A15	7,8	13,1	5,5	5,3	6,7	3,7	5,9	7,4	15	12,8	13,9	2,2	3,8	7,3	7,3	4,5	3,7	13
A16	16,2	14,9	10,6	9,3	7,5	10,3	7,4	7,4	2,7	3,9	4,4	12,1	10,5	9,7	10,8	13,9	15,4	19
A17	16,2	15,5	10,6	9,1	7,5	10	7,2	7,5	4	5,1	5,8	11,7	10,2	9,9	11	13,7	15	19,4
A18	0	4,4	5,5	6,8	8,6	5,9	8,8	8,8	17,5	14,2	15	4,3	5,7	6,6	5,6	2,2	1,1	1,8
A19	4,4	0	2	3,5	5,2	2,7	5,5	5,3	13,9	10,4	11,1	1,2	2,5	3	1,9	2,6	4,3	5,5
A20	5,5	2	0	1,3	3,1	0,4	3,2	3,3	12,1	8,8	9,6	1,4	0,2	1,2	0,1	3,3	4,7	7,4
A21	6,8	3,5	1,3	0	1,8	1	1,9	2	10,8	7,5	8,3	2,7	1,2	0,2	1,3	4,6	6	8,8
A22	8,6	5,2	3,1	1,8	0	2,8	0,1	0,2	9	5,7	6,5	4,5	3	2	3	6,4	7,8	10,5
A23	5,9	2,7	0,4	1	2,8	0	2,9	3	11,8	8,6	9,4	1,7	0,2	0,9	0,4	3,7	5	8
A24	8,8	5,5	3,2	1,9	0,1	2,9	0	0,1	8,9	5,7	6,5	4,6	3	2,1	3,2	6,5	7,8	10,8
A25	8,8	5,3	3,3	2	0,2	3	0,1	0	8,8	5,4	6,2	4,8	3,2	2,2	3,2	6,7	8,1	10,6
A26	17,5	13,9	12,1	10,8	9	11,8	8,9	8,8	0	3,4	2,6	13,6	12	10,9	11,9	15,4	16,9	19,3
A27	14,2	10,4	8,8	7,5	5,7	8,6	5,7	5,4	3,4	0	0,8	10,3	8,7	7,6	8,6	12,2	13,7	15,9
A28	15	11,1	9,6	8,3	6,5	9,4	6,5	6,2	2,6	0,8	0	11,2	9,6	8,4	9,4	13	14,5	16,6
A29	4,3	1,2	1,4	2,7	4,5	1,7	4,6	4,8	13,6	10,3	11,2	0	1,5	2,7	1,7	2	3,3	6,5
A30	5,7	2,5	0,2	1,2	3	0,2	3	3,2	12	8,7	9,6	1,5	0	1,1	0,2	3,5	4,8	7,8
A31	6,6	3	1,2	0,2	2	0,9	2,1	2,2	10,9	7,6	8,4	2,7	1,1	0	1	4,5	6	8,4
A32	5,6	1,9	0,1	1,3	3	0,4	3,2	3,2	11,9	8,6	9,4	1,7	0,2	1	0	3,5	5	7,3
A33	2,2	2,6	3,3	4,6	6,4	3,7	6,5	6,7	15,4	12,2	13	2	3,5	4,5	3,5	0	1,3	4,2
A34	1,1	4,3	4,7	6	7,8	5	7,8	8,1	16,9	13,7	14,5	3,3	4,8	6	5	1,3	0	3,3
A35	1,8	5,5	7,4	8,8	10,5	8	10,8	10,6	19,3	15,9	16,6	6,5	7,8	8,4	7,3	4,2	3,3	0

Figure 2. Distance Matrix

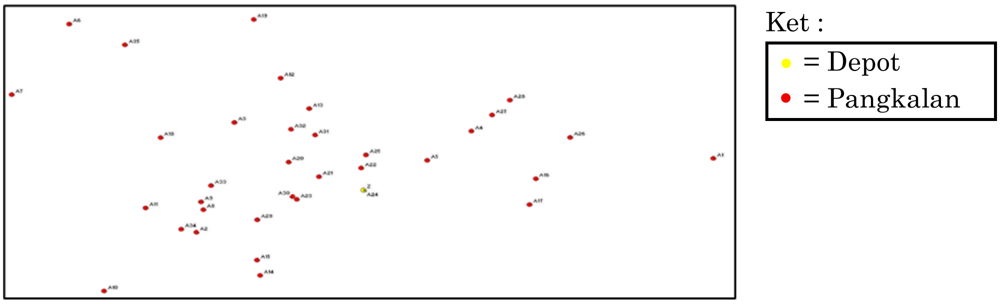

Figure 3. Base Distance Distribution

3.4 Initial Route Calculation of PT. Cahaya Mentari Bumi Perkasa

Table 4. Route of Initial Distribution

Fleet Utilized	Route of Delivery	Distance Traveled (km)	Payload (Cylinders)	Completion Time (Menit)
Truk 1	Z→A24→A22→A25→A21→A31→Z	6,3	558	99,3
Truk 2	Z→A23→A30→A20→A32→A13→Z	9,8	482	90,1
Truk 3	Z→A5→A4→A27→A28→A6→ Z	43	559	136,2
Truk 1	Z→A29→A19→A33→A9→ Z	16,6	559	109,8
Truk 2	Z→A15→A14→A2→A34→ Z	19,1	500	102,4
Truk 3	Z →A3→A12→A18→A35→ Z	30,5	337	86,7
Truk 1	Z→A8→A11→A16→17→Z	34,7	455	110,5
Truk 2	Z→A7→A10 → Z	39,7	214	75,4
Truk 3	Z→A26→A1→Z	30,6	196	63,3
	Total	**230,3**	**3860**	**873,6**

3.5 Distance Calculation with Cheapest Insertion Heuristics Algorithm

The determination of the LPG gas distribution route employs the Cheapest Insertion Heuristics method, utilizing an insertion strategy to establish an initial route commencing from the base (Z) to the nearest point (Z→A22), which has a request for 119 cylinders. Subsequently, generate a new insertion for each combination external to the subtour. Route insertion ceases upon reaching fleet capacity. The insert value $c_{ik}+c_{kj}-c_{ij}$, to c_{ik} = distance i to k, c_{kj} = distance k to j, c_{ij} = distance i to j

Table 5. Initial Calculation of the Cheapest Insertion Heuristics Algorithm

Arc Scheduled for Replacement	Arc Added to Subtour	Additional Distance	Arc yang Akan Diganti	Arc Scheduled for Replacement	Additional Distance
(Z,A22)	(Z,A1)→(A1,A22)	$C_{Z,A1}+C_{A1,A22}-C_{Z,A22}$ = 28,9	(Z,A22)	(Z,A18)→(A1,A22)	$C_{Z,A18}+C_{A18,A22}-C_{Z,A22}$ = 16,5
(Z,A22)	(Z,A2)→(A1,A22)	$C_{Z,A2}+C_{A2,A22}-C_{Z,A22}$ = 14,2	(Z,A22)	(Z,A19)→(A1,A22)	$C_{Z,A19}+C_{A19,A22}-C_{Z,A22}$ = 14,4
(Z,A22)	(Z,A3)→(A1,A22)	$C_{Z,A3}+C_{A3,A22}-C_{Z,A22}$ = 11,4	(Z,A22)	(Z,A20)→(A1,A22)	$C_{Z,A20}+C_{A20,A22}-C_{Z,A22}$ = 5,4
(Z,A22)	(Z,A4)→(A1,A22)	$C_{Z,A4}+C_{A4,A22}-C_{Z,A22}$ = 9,5	(Z,A22)	(Z,A21)→(A1,A22)	$C_{Z,A21}+C_{A21,A22}-C_{Z,A22}$ = 2,6
(Z,A22)	(Z,A5)→(A1,A22)	$C_{Z,A5}+C_{A5,A22}-C_{Z,A22}$ = 4,8	(Z,A22)	(Z,A23)→(A1,A22)	$C_{Z,A23}+C_{A23,A22}-C_{Z,A22}$ = 4,5
(Z,A22)	(Z,A6)→(A1,A22)	$C_{Z,A6}+C_{A6,A22}-C_{Z,A22}$ = 29,1	(Z,A22)	(Z,A24)→(A1,A22)	$C_{Z,A24}+C_{A24,A22}-C_{Z,A22}$ = 1
(Z,A22)	(Z,A7)→(A1,A22)	$C_{Z,A7}+C_{A7,A22}-C_{Z,A22}$ = 30,2	(Z,A22)	(Z,A25)→(A1,A22)	$C_{Z,A25}+C_{A25,A22}-C_{Z,A22}$ = 0,9
(Z,A22)	(Z,A8)→(A1,A22)	$C_{Z,A8}+C_{A8,A22}-C_{Z,A22}$ = 12,9	(Z,A22)	(Z,A26)→(A1,A22)	$C_{Z,A26}+C_{A26,A22}-C_{Z,A22}$ = 17,1
(Z,A22)	(Z,A9)→(A1,A22)	$C_{Z,A9}+C_{A9,A22}-C_{Z,A22}$ = 12,9	(Z,A22)	(Z,A27)→(A1,A22)	$C_{Z,A27}+C_{A27,A22}-C_{Z,A22}$ = 11,3
(Z,A22)	(Z,A10)→(A1,A22)	$C_{Z,A10}+C_{A10,A22}-C_{Z,A22}$ = 24,1	(Z,A22)	(Z,A28)→(A1,A22)	$C_{Z,A28}+C_{A28,A22}-C_{Z,A22}$ = 13,2
(Z,A22)	(Z,A11)→(A1,A22)	$C_{Z,A11}+C_{A11,A22}-C_{Z,A22}$ = 17,7	(Z,A22)	(Z,A29)→(A1,A22)	$C_{Z,A29}+C_{A29,A22}-C_{Z,A22}$ = 8,1
(Z,A22)	(Z,A12)→(A1,A22)	$C_{Z,A12}+C_{A12,A22}-C_{Z,A22}$ = 11,8	(Z,A22)	(Z,A30)→(A1,A22)	$C_{Z,A30}+C_{A30,A22}-C_{Z,A22}$ = 4,8
(Z,A22)	(Z,A13)→(A1,A22)	$C_{Z,A13}+C_{A13,A22}-C_{Z,A22}$ = 7,7	(Z,A22)	(Z,A31)→(A1,A22)	$C_{Z,A31}+C_{A31,A22}-C_{Z,A22}$ = 4,4
(Z,A22)	(Z,A14)→(A1,A22)	$C_{Z,A14}+C_{A14,A22}-C_{Z,A22}$ = 12,5	(Z,A22)	(Z,A32)→(A1,A22)	$C_{Z,A32}+C_{A32,A22}-C_{Z,A22}$ = 6,4
(Z,A22)	(Z,A15)→(A1,A22)	$C_{Z,A15}+C_{A15,A22}-C_{Z,A22}$ = 11,4	(Z,A22)	(Z,A33)→(A1,A22)	$C_{Z,A33}+C_{A33,A22}-C_{Z,A22}$ = 11,8
(Z,A22)	(Z,A16)→(A1,A22)	$C_{Z,A16}+C_{A16,A22}-C_{Z,A22}$ = 13,7	(Z,A22)	(Z,A34)→(A1,A22)	$C_{Z,A34}+C_{A34,A22}-C_{Z,A22}$ = 14,7
(Z,A22)	(Z,A17)→(A1,A22)	$C_{Z,A17}+C_{A17,A22}-C_{Z,A22}$ = 13,4	(Z,A22)	(Z,A35)→(A1,A22)	$C_{Z,A35}+C_{A35,A22}-C_{Z,A22}$ = 22,2

The outcome of the minimal weight augmentation of the insertion (Z,A22) is 1.24 km. The arc (Z,A22) is substituted by (Z,A25)→(A25,A22), resulting in a total need of 302 tubes. The insertion continues until the capacity is reached without surpassing the maximum limit.

Table 6. Route with *Cheapest Insertion Heuristics Algorithm*

Fleet Utilized	Route of Delivery	Distance Traveled (km)	Payload (Tabung)	Completion Time (Menit)
Truk 1	Z→A24→A25→A22→A35→A11→Z	29,2	512	1752
Truk 2	Z→A13→A12→A3→A18→A33→Z	22,2	541	1332
Truk 3	Z→A7→A6→A10→A17→ A16→Z	49	519	2940
Truk 1	Z→A5→ Z	6,4	153	384
Truk 2	Z→A21→A31→A19→A20→A23→Z	10,5	548	630
Truk 3	Z→A14→A15→A8→A9→ Z	18,2	529	1092
Truk 1	Z→A30→A32→A29→A2→ A34→Z	16,5	544	990
Truk 2	Z→A27→A28→A26→A1→ A4→Z	32,6	543	1956
	Total	**184,6**	**3860**	**110,76**

The path derived by the Cheapest Insertion Heuristics algorithm is more efficient than the original 230,3 kilometer route. Nonetheless, a single route exclusively supplies one depot, leading to imbalanced delivery among fleets. Consequently, the Cheapest Insertion Heuristics algorithm can be enhanced by Dynamic Programming to optimize delivery to LPG depots.

3.6 Dynamic Programming for Distance Calculation

Dynamic Programming arose from the inclination to examine and display computational outcomes at each level via tables, enabling detailed calculation and identification of solutions(Elsa et al., 2023).

The models:

$$f_n(s) = \min\{ f_n(x_n) + f_{n+1}(x_{n+1}) \} \tag{2}$$

Details: $f_n(s)$ = purchase value at stage n x_n = best route at stage n

s = distance

Table 7. Calculation Results with Dynamic Programming

Fleet Utilized	Route of Delivery	Distance Traveled (km)	Payload (Tabung)	Completion Time (Menit)
Truk 1	Z→A24→A25→A22→A21→A31→Z	6,1	558	99,1
Truk 2	Z→A23→A30→A32→A20→A29→Z	9,6	496	92,3
Truk 3	Z→A5→A4→A27→A28→ Z	15,6	500	98,9
Truk 1	Z→A13→A12→A3→A18→ Z	22,6	363	83,1
Truk 2	Z→A15→A14→A2→A34→ Z	19,1	500	102,4
Truk 3	Z→A33→A9→A8→A11→ Z	20,7	535	109,9
Truk 1	Z→A17→A16→A26→A1→ Z	32,5	442	106,2
Truk 2	Z→A19→A35→A6→A7→ A10→Z	32,5	466	110,2
	Total	**158,7**	**3860**	**802**

Employing Dynamic Programming for route optimization based on the initial results of the Cheapest Insertion Heuristic produces a more efficient and equitable route, resulting in a total travel distance of 158.7 km and a uniform distribution of fleet load. This efficiency arises from DP's capacity to formulate optimal subroutes and more distance-efficient visitation sequences. In contrast to the study by (Triyanto et al., 2019) which documented a 15% savings, the CIH-DP approach is demonstrated to be more efficient for medium-scale distribution. However, changes in demand can influence the route's optimality due to the heuristic characteristics of CIH being responsive to input variations. Consequently, periodic reoptimization or dynamic algorithms are advised. The CIH-DP method is appropriate for execution by PT. Cahaya Mentari Bumi Perkasa (Table 7) is effective in minimizing expenses and adhering to delivery schedules.

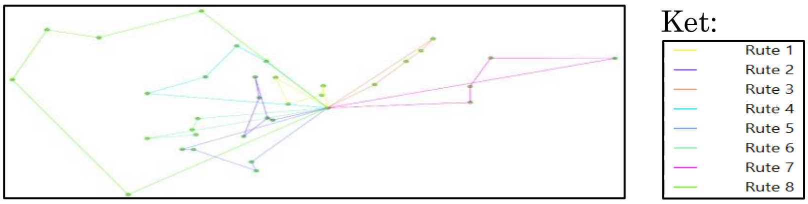

Figure 4. Optimal Route of Dynamic Programming

3.7 Distribution Cost Calculation

The distribution utilizes subsidized diesel fuel. Each canter truck can cover an average distance of around 8 kilometers per liter. The cost of diesel fuel is approximately Rp. 6,800/liter. Cost assessments are conducted on the original route, the route produced using the Cheapest Insertion Heuristic Algorithm, and Dynamic Programming. This study calculates distribution costs, encompassing both fixed and variable expenses. Fixed costs encompass maintenance, administration, and taxes associated with each vehicle for a single journey. Variable expenditures encompass fuel expenses for each delivery and oil changes conducted every three months(Tarnoto et al., 2021).

Table 8. Distribution Costs

Condition	Distance Traveled (km) (A)	*Fixed Cost*	*Variable Cost* (A x Rp. 8.233/Km)	Total (Rp)
Rute Awal Distribusi	230,3	Rp2.394.000	Rp1.896.060	Rp4.290.060
Dynamic Programming	158,7	Rp2.128.000	Rp1.306.577	Rp3.434.577

The calculated distribution costs will yield the ideal path for LPG gas delivery at PT. Cahaya Mentari Bumi Perkasa.

Table 9. Optimal Route for LPG Gas Delivery

Condition	Tour	Route	Payload Utility	Total Distance	Total Time	Total Distribution Costs
Initial Distribution Route	1	Z→A24→A22→A25→A21→A31→Z	99,6%	6,3	99,3	Rp4.290.060
	2	Z→A23→A30→A20→A32→A13→Z	86,1%	9.8	90,1	
	3	Z→A5→A4→A27→A28→A6→ Z	99,8%	43	136,2	
	4	Z→A29→A19→A33→A9→ Z	99,8%	16,6	109,8	
	5	Z→A15→A14→A2→A34→ Z	89,3%	19,1	102,4	
	6	Z →A3→A12→A18→A35→ Z	60,2%	30,5	86,7	
	7	Z→A8→A11→A16→17→Z	81,3%	34,7	110,5	
	8	Z→A7→A10 → Z	38,2%	39,7	75,4	
	9	Z→A26→A1→Z	35%	30,6	63,3	
		Total		**220,5**	**873,6**	
Dynamic Programming	1	Z→A24→A25→A22→A21→A31→Z	99,6%	6,1	99,1	Rp3.434.577
	2	Z→A23→A30→A32→A20→A29→Z	88,6%	9,6	92,3	
	3	Z→A5→A4→A27→A28→ Z	89,3%	15,6	98,9	
	4	Z→A13→A12→A3→A18→ Z	64,8%	22,6	83,1	
	5	Z→A15→A14→A2→A34→ Z	89,3%	19,1	102,4	
	6	Z→A33→A9→A8→A11→ Z	95,5%	20,7	109,9	
	7	Z→A17→A16→A26→A1→ Z	78,9%	32,5	106,2	
	8	Z→A19→A35→A6→A7→ A10→Z	83,2%	46,9	110,2	
		Total		**158,7**	**802**	
		Selisih		**61,8**	**71,6**	**Rp855.483**
		Persentase Penghematan		**28%**	**8,2%**	**19,9%**

4. CONCLUSION

This research illustrates that the efficiency of LPG distribution at PT Cahaya Mentari Bumi Perkasa can be enhanced through the combined implementation of the Cheapest Insertion Heuristic and Dynamic Programming algorithms. This approach effectively decreased travel distance by 28%, travel time by 8.2%, and distribution costs by 19.9%. Additionally, it reduced the number of journeys from 9 to 8, which led to an increase in fleet efficiency. Academically, this research contributes to the advancement of a hybrid approach that integrates the precision of dynamic programming with the speed of heuristic algorithms to solve the Capacitated Vehicle Routing Problem (CVRP). Consequently, it can be effectively implemented to enhance the efficiency of long-term distribution in organizations. Nevertheless, the research's limitations are in its static assumptions regarding demand and its neglect of dynamic variables, including time, traffic, and weather. The flexibility and accuracy of distribution route planning can be enhanced by the construction of an adaptive model that integrates spatial-temporal data and metaheuristic algorithms.

6. RECOMMENDATIONS

This study improves VRP optimization by enhancing route accuracy and contextual relevance. Future research may explore different locations and compare other heuristic or metaheuristic algorithms to identify more optimal distribution paths.

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